Department of Mathematics
Minor 2: MTL 100 Calculus
Date: 18 August 2020

$4 \times 5 = 20$

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1. Find the nature (locl max, local min, saddle point) of critical points of the function
$$f(x, y) = 2x^2 - 16xy + 4y^4 + 2.$$

2. Consider the problem:
maximize/minimize: $2x^2 + 3y^2 + z^2$ subject to the constraint $3x - 4y + 5z = 1$.
   (a) Explain why there is no maximum value possible.
   (b) Use the Lagrange multiplier method to find (x_0, y_0, z_0) where the minimum value is attained.

3. Let $f, g : \mathbb{R} \longrightarrow \mathbb{R}$ be two functions satisfying
$$f(x + y) = f(x)g(y) + g(x)f(y) \quad \text{and} \quad g(x + y) = g(x)g(y) - f(x)f(y).$$
If f and g are differentiable at 0 show that f and g are differentiable at every $c \in \mathbb{R}$.

4. Consider a function $f(x)$ whose second derivative $f''(x)$ exists and is continuous on $[0, 1]$. Assume that $f(0) = f(1)$ and suppose there exists $A > 0$ such that $|f''(x)| \leq A$. Show that
$$|f'(1/2)| \leq \frac{A}{4} \quad \text{and} \quad |f'(x)| \leq \frac{A}{2}, \text{ for } 0 < x < 1.$$
(Hint: Use Taylor's theorem)

GOOD LUCK

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