

Major Test:: MAL 101::: November 2013

There are ten questions. Every question is compulsory. Marks will not be awarded if appropriate arguments are not provided.

Maximum Marks: 50

Maximum Time: Two hours

- (1) Suppose $\rho_1, \rho_2, \dots, \rho_n$ are elementary row operations such that $\rho_n \circ \rho_{n-1} \circ \dots \circ \rho_1(A) = I$ for a square matrix A (here, I is the identity matrix). Describe the inverse of A using these row operations. Justify. Use this to find the inverse of the following matrix (indicate the row operations applied in each step). [5 = 2 + 3]

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

- (2) Suppose \mathcal{P}_4 is the real vector space $\{a + bx + cx^2 + dx^3 : a, b, c, d \in \mathbb{R}\}$. Suppose $D : \mathcal{P}_4 \rightarrow \mathcal{P}_4$ is the derivative operator [5 = 3 + 1 + 1]

$$D(a + bx + cx^2 + dx^3) = b + 2cx + 3dx^2.$$

Let $T = D^2 - D$. Find the null space of T and $\text{rank}(T)$. What are all the eigenvalues of T ? Write eigen-space corresponding to each eigenvalue.

- (3) Prove the following statements:

$$[5 = 2 + 3]$$

(a) If W_1 and W_2 are subspaces of a vector space V over \mathbb{R} then $W_1 + W_2$ is the span of $W_1 \cup W_2$.

(b) If $\{u + v, v + w, w + u\}$ is a linearly independent subset of a vector space V then $\{u, v, w\}$ is linearly independent.

- (4) Describe Picard iteration method to find approximate solutions of an initial value problem (of a first order ODE). Find the first three iterations y_1, y_2, y_3 of the following IVP using Picard method. [5 = 1 + 2 + 2]

$$y' - xy = x, \quad y(0) = 0.$$

What is y_n for $n \in \mathbb{N}$? Justify it by the method of mathematical induction (i.e., assuming the formula for n , prove it for $n + 1$).

- (5) Solve the following second order ODE using the method of undetermined coefficients. [5]

$$y'' + y = 2 \cos x, \quad y(0) = 1, \quad y'(0) = 1.$$

PTO

$y_1 = 1 + \cos x$
 $y_2 = 1 + \sin x$
 $y_3 = 1 + \cos x$
 $y_4 = 1 + \sin x$
 $y_5 = 1 + \cos x$

$y = 1 + \cos x + \sin x$
 $y' = -\sin x + \cos x$
 $y(0) = 1 + 1 = 2$
 $y'(0) = 0 + 1 = 1$