Major Test::: MAL 101::: May 2014

Marks will not be awarded if appropriate arguments are not provided.

Maximum Marks: **50** Maximum Time: **Two hours**

(1) Describe all the elementary row operations along with their inverses. Find the dimension of the subspace of $\mathbb{R}^4$ spanned by the following set [5 = 3 + 2]

$$\{(1,1,1,1), (1,2,3,4), (-3,-2,1,1), (6,6,4,5)\}.$$ 4

(2) Suppose $T : V \to V$ is a linear operator satisfying $T^2 = 0$. Show that [3 = 1 + 2]
 (a) T is not invertible.
 (b) if V is finite dimensional, $\text{rank}(T) \leq \frac{1}{2}\dim V$. 1

(3) Find the dimensions of the two proper subspaces of $\mathbb{R}^4$ if their union spans whole of $\mathbb{R}^4$ and the intersection is a straight line passing through the origin. [3] 2

(4) Let $B = \{1, 1+X, 1+X+X^2\}$, $\hat{B} = \{1, 1-X, 1-X+X^2\}$. Observe (but do not prove) that $B, \hat{B}$ are bases of $\mathcal{P}_3 = \{a + bX + cX^2 : a, b, c \in \mathbb{R}\}$. Suppose $v \in \mathcal{P}_3$ is such that the coordinate vector $[v]_B$ of v with respect to B is the column $(1\ 1\ 1)^T$. Find the coordinate vector $[v]_{\hat{B}}$ of v with respect to $\hat{B}$. [3] 3

(5) Solve the following IVP (write y explicitly as a function of x): [4]

$$\frac{dy}{dx} - xy = y^3 e^{-(x+1)^2}, \quad y(0) = \beta\ (>0).$$ 4

(6) Suppose p and q are continuous functions on an open interval I. Let y_1 and y_2 be solutions of $y'' + p(x)y' + q(x)y = 0$ defined on I. Show that y_1 and y_2 are linearly dependent if their Wronskian $W(y_1, y_2)$ is zero for some $x_0 \in I$. Further, show that if $W(y_1, y_2)$ is zero at $x_0 \in I$, then it is identically zero on I. [5 = 4 + 1] $\boxed{20}$ 3

(7) Solve the following ODE using the method of undetermined coefficients: [5]

$$y'' - 4y' + 4y = 2e^{2x}, \quad y(0) = 1, \ y'(0) = 3.$$ 5

(8) Using the method of variation of parameters find the general solution: [5]

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t}.$$ 2

(9) Find the power series solution of following ODE: [4]

$$y'' + x^3 y = 0.$$ 1

Further, calculate the first seven coefficients of the series if $y(0) = 1$, $y'(0) = 1$.

(10) Find the eigenvalues and eigenfunctions of the following Stum-Liouville boundary value problem: [5]

$$y'' + 4y' + (\lambda + 4)y = 0, \quad y(0) = 0, \ y(\pi) = 0.$$ 4

(11) Let δ be the Dirac delta. Using Laplace transform solve [4]

$$4y'' + 4y' + 5y = \delta(t-1), \quad y(0) = 0, \ y'(0) = 1.$$ 8

(12) Using Laplace transform, find y satisfying the following integral equation: [4]

$$y(t) + 2\int_0^t \cos(s) y(t-s)\, ds = \cos t.$$

:::END:::