MTL101::: Major Test

November, 2014

1. Total mark : 45, Maximum Time : 2 hours.
2. No marks will be awarded if appropriate justification is not provided.

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1. Consider the following IVP [4 = 2 + 2]

$$y' = \sqrt{t} + y^2, \quad y(0) = y_0 = 0.$$

(a) Discuss the existence and uniqueness of this IVP.

(b) Find the first two Picard's iterations y_1 and y_2.

2. Find the general solution of [5]

$$t^2y'' - 4ty' + 6y = \sin(\ln t).$$

3. Solve the following IVP [5]

$$y'' - 4y' + 4y = \delta(t-2) + H_1(t), \quad y(0) = y'(0) = 0,$$

where δ and H are Dirac-delta function and Heaviside function respectively.

4. Find the general solution of $\vec{x}\,' = A\vec{x}$, where $A = \begin{pmatrix} 8 & 12 & -2 \\ -3 & -4 & 1 \\ -1 & -2 & 2 \end{pmatrix}$. [6]

5. Let [5]

$$f(t) = \mathcal{L}^{-1}\left[\frac{e^{-\pi s/2} - e^{-3\pi s/2}}{s(s^2+2s+5)}\right],$$

where $\mathcal{L}^{-1}$ is the inverse Laplace transform. Find the value of $f(2\pi)$.

6. Find the general solution of the following system of differential equations: [5]

$$\frac{dx}{dt} = 2x + 6y + e^t$$

$$\frac{dy}{dt} = x + 3y - e^t$$

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7. Apply elemetary row operations on the augmented matrix of the given system of linear non-homogenous equations so that the coefficient matrix is a row reduced echelon matrix. Conclude that there are infinitely many solutions and find the general solution by assigning arbitrary value(s) to the free unknown(s). [4]

$$\begin{aligned} x + 2y + 3z + w &= 4 \\ 2x + 3y + z - w &= 4 \\ 3x + 4y - z + 2w &= 5 \\ 2x + 3y + z + 4w &= 5 \end{aligned}$$

8. Let [4]

$$W_1 = \{(x, y, z, w) \in \mathbb{R}^4 : x = 0, y + z + w = 0\}$$

and

$$W_2 = \{(x, y, z, w) \in \mathbb{R}^4 : x - y = 0, x + y + z + w = 0\}.$$

Find a basis B_1 of $W_1 \cap W_2$ and find a basis B_2 of $W_1 + W_2$ containing B_1.

9. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation. Suppose [4]

$$\begin{aligned} T(1, 1, 1) &= (1, -1, 0), \\ T(1, -1, 1) &= (0, 1, -1), \\ T(0, 1, -1) &= (-1, 0, 1). \end{aligned}$$

Find the nullity of T and the vector $T(0, 0, 1)$.

10. Find the span of the set

$$\left\{ \begin{pmatrix} 1 & t \\ t^2 & t^3 \end{pmatrix} \in M_2(\mathbb{R}) : t \in \mathbb{R} \right\}$$

in the vector space $M_2(\mathbb{R})$ over $\mathbb{R}$. [3]

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