MTL 101
LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

MAJOR: PART A

Notation: $x'(t), x''(t)$ and $x'''(t)$ denote the first, second and third derivatives of function x with respect to variable t, respectively.

PART A: 10:00 to 11:20.

No Submission is allowed after 11:20.

If you submit PART A with PART B, it will not be graded.

Question 1: (3 Marks) Find the orthogonal trajectories of the family of curves $x = ce^{t^2}$.

Question 2: (5 Marks) Consider the IVP

$$\frac{dx}{dt} = \frac{\cos x}{1 - t^2}, \quad x(0) = x_0,$$

with $x_0 \in \mathbb{R}$. Find the largest interval $|t| < h$ on which the existence and uniqueness theorem guarantees a unique solution.

Question 3: (4 Marks) Solve the following ODE

$$(xdt + tdx)\, t \cos\left(\frac{x}{t}\right) = (tdx - xdt)\, x \sin\left(\frac{x}{t}\right).$$

Question 4: (6 Marks) Consider the differential operator with constant coefficients

$$L = \frac{d^2}{dt^2} - 2\frac{d}{dt} + 1$$

and let $f(t) = \sin(t) + e^t + t$.

(a) Determine M, a linear differential operator with constant coefficients, such that it annihilates f.

(b) Taking into account **part(a)**, find the general solution of $L(x(t)) = f(t)$ using the method of undetermined coefficients.

Question 5: (8 Marks) Consider the differential equation

$$t^3 x''' + t^2 x'' - 2x = 1 \tag{1}$$

(a) Reduce (1) into an ODE with constant coefficients and find the corresponding first order system.

(b) Find the fundamental matrix of the first order system obtained in **part(a)**.

(c) Using the method of variation of parameters for the first order system obtained in **part(a)**, find the general solution of (1).

END OF PART A