

MTL 101
LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
MAJOR: PART B

Notation: $x'(t)$, $x''(t)$ and $x'''(t)$ denote the first, second and third derivatives of function x with respect to variable t , respectively.

PART B: 11:45 to 13:05.
No Submission is allowed after 13:05.

DO NOT UPLOAD PART A WITH PART B. IT WILL NOT BE GRADED.

Question 1: (6 Marks) Discuss the existence and uniqueness of the solutions for the following IVPs:

(a)

$$\begin{aligned}x_1' &= |x_1|^p + t^2, \quad p \geq 1 \\x_2' &= \tan^{-1}(x_2), \\x_1(1) &= 0, \quad x_2(1) = 0.\end{aligned}$$

(b)

$$\begin{aligned}x_1' &= |x_2|^p + t^2, \quad 0 < p < 1 \\x_2' &= \sin(x_1), \\x_1(1) &= 0, \quad x_2(1) = 0.\end{aligned}$$

Question 2: (5 Marks) Solve the following IVP using Laplace Transform:

$$x'' + 3x' + 2x = \sin(t) + \delta(t - 2)$$

with initial conditions $x(0) = 0$ and $x'(0) = 5$. Here $\delta(t - 2)$ is the Dirac Delta function centred at $t = 2$. Check if the solution is continuously differentiable (C^1).

Question 3: (6 Marks) Find the inverse Laplace Transform of the following functions:

$$(a) \ln\left(1 - \frac{a^2}{s^2}\right), \quad (b) \frac{s^2}{(s^2 + a^2)^2}.$$

Question 4: (7 Marks) Consider the following linear system of equations:

$$\begin{aligned}x_1' &= -2x_1 + x_2 + 4x_3 \\x_2' &= -5x_1 + 2x_2 + 5x_3 \\x_3' &= -x_1 + x_2 + \lambda x_3\end{aligned}$$

If $\vec{x} = \vec{a}te^{2t}$ is a solution of this system for some constant vector \vec{a} , then find the general solution of the system.

END OF PART B