MINOR TEST 1 (MTL101)

Maximum Time: 1 Hour Max Marks: 20

All questions carry equal marks

1. Let V and U be vector spaces over a field $\mathbb{F}$. If $\phi : V \to \cancel{W}\ U$ is a vector space homomorphism of V onto U, with $Kernel(\phi) = W$, then prove that V/W is isomorphic onto U.

2. Let $T : \mathbb{R}^4{}_{\mathbf{R}} \to \mathbb{R}^3{}_{\mathbf{R}}$ be the map $(x_1, x_2, x_3, x_4) \to (x_1 - x_4, x_2 + x_3, x_3 - x_4)$.

   (a) Prove that T is a homomorphism of $\mathbb{R}^4{}_{\mathbf{R}}$ onto $\mathbb{R}^3{}_{\mathbf{R}}$.

   (b) Determine the kernel and range of T and compute their dimensions over $\mathbb{R}$.

3. (a) Let V and W be vector spaces over a field $\mathbb{F}$. If vectors $v_1, v_2, \cdots, v_n \in V$ are linearly independent over $\mathbb{F}$ and if $T : V \to W$ is a one to one homomorphism of V into W, mapping v_i to w_i for each $i = 1, \cdots, n$, then prove that $w_1, w_2, \cdots, w_n$ are linearly independent vectors in W over $\mathbb{F}$.

   (b) Determine the linear span $L(S)$ over $\mathbb{R}$ for $S = \{(2, 3, -4), (-4, -5, 8), (8, 2, -16)\}$. Also determine its dimension over $\mathbb{R}$.

4. Let V be the vector space of all polynomials $p(x) \in \mathbb{R}[x]$ of degree at most 2 over the field $\mathbb{R}$. Let $T : V \to V$ be the map sending $p(x)$ to $p(x) + p'(0)$, $\forall\ p(x) \in V$. Here, $p'(x)$ is the derivative of $p(x)$ w.r.t. x.

   (a) Prove that T is a homomorphism.

   (b) Is T onto V? Justify.