Indian Institute of Technology Delhi
MTL 101 (Minor Test 1)
August 2015

Max Time: 1 hour | Max Marks: 20

Note: No marks will be awarded without appropriate arguments.

1. Consider the following system of linear equations:

$$
\begin{aligned}
x_1 + x_2 + x_3 + x_4 + x_5 &= 0 \\
x_1 - x_2 + 3x_3 - x_4 + x_5 &= 0 \\
x_1 + 2x_2 + 3x_4 &= 0 \\
3x_1 + 2x_2 + 4x_3 + 3x_4 + 2x_5 &= 0
\end{aligned}
$$

   (a) Find all the solutions by reducing the coefficient matrix to its RRE form.

   (b) Find a basis for the solution space of the above system. [3+2=5]

2. Let W_1 and W_2 be two subspaces of $\mathbb{R}^4$ defined by

$$
W_1 = \{(x,y,z,w) \in \mathbb{R}^4 : x+y+z+w=0, y+z=0\},
$$
$$
W_2 = \{(x,y,z,w) \in \mathbb{R}^4 : x+y+z=0, y+z+w=0\}.
$$

   (a) Find a basis for $W_1 \cap W_2$.

   (b) Find a basis for $W_1 + W_2$. [2+2=4]

3. Let $\mathcal{B}_1 = \{(1,0,0),(0,1,0),(0,0,1)\}$ and $\mathcal{B}_2 = \{(1,0,0),(1,0,1),(1,1,1)\}$ be two *ordered* bases for $\mathbb{R}^3$.

   (a) Find the matrix P such that $[v]_{\mathcal{B}_2} = P[v]_{\mathcal{B}_1}$.

   (b) Find the coordinate vector $[(2,-3,1)]_{\mathcal{B}}$ using part (a). [2+2=4]

4. (a) Let $A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \in M_3(\mathbb{R})$ and let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined as

$$
T(x,y,z) = (a_1x + a_2y + a_3z,\ b_1x + b_2y + b_3z,\ c_1x + c_2y + c_3z).
$$

   Prove that if T is injective (i.e., one-to-one) then $\det(A) \neq 0$.

   (b) Find a linear transformation $S : \mathbb{R}^3 \to \mathbb{R}^3$ such that $\text{nullity}(S) = 1$. [2+2=4]

5. Let B be a subset of a vector space V such that B spans V but no proper subset of B spans V. Prove that B is a basis for V. [3]

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