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2017CS10328

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MTL 101 Linear Algebra and Differential Equations: Minor-I

Total marks: 20 Time: 1 hour

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1. Every question is compulsory
2. No marks will be provided if appropriate justification is not provided

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1. Determine (with justification) whether the following statements are true or false. [6]

   (a) If in a vector space V, $W_1 \oplus W_2 = W_1' \oplus W_2'$ and $W_1 = W_1'$, then $W_2 = W_2'$.

   (b) If B is an ordered basis of a finite dimensional vector space V and $u, v \in V$, then we have
   $$[u]_B = [v]_B \text{ implies } u = v.$$

   (c) If X_1, X_2 and X_3 are solutions of the system $AX = B$, then $X_1 - 2X_2 + X_3$ is NOT a solution of $AX = B$, where $A \in M_{m\times n}(\mathbb{R})$ and $B \in M_{m\times 1}(\mathbb{R})$, $B \neq 0$.

2. Suppose that $\{v_1, v_2, v_3, v_4\}$ is a linearly independent subset of a vector space V. Is the set $\{v_1 + 2v_2 + v_3 + 2v_4,\ v_1 - v_2 - v_3 + v_4,\ 2v_1 - 5v_2 - 4v_3 + v_4,\ 4v_1 + 2v_2 + 6v_4\}$ linearly independent? [3]

3. (a) Show that a minimal spanning set is a basis. [2]

   (b) Give an example of a vector space V and two subspaces W_1 and W_2 of V satisfying the following two conditions: [3]

      i. W_1 is non-zero and finite dimensional,

      ii. W_2 is infinite dimensional, $W_2 \neq V$ and $W_1 \subset W_2$.

4. Let
$$W_1 = \left\{ \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix} \in M_{3\times3}(\mathbb{R}) : \begin{array}{r} x_1 + x_2 + x_3 = 0, \\ y_1 + y_2 + y_3 = 0, \\ z_1 + z_2 + z_3 = 0 \end{array} \right\}$$
and
$$W_2 = \left\{ \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix} \in M_{3\times3}(\mathbb{R}) : \begin{array}{r} x_1 + y_1 + z_1 = 0, \\ x_2 + y_2 + z_2 = 0, \\ x_3 + y_3 + z_3 = 0 \end{array} \right\}.$$

   (a) Find $\dim(W_1 \cap W_2)$. [3]

   (b) Find $\dim(W_1 + W_2)$. [2]

   (c) Show that $W_1 + W_2 = \left\{ A = [a_{ij}] \in M_{3\times3}(\mathbb{R}) : \sum_{j=1}^{3} \sum_{i=1}^{3} a_{ij} = 0 \right\}$. [1]