Department of Mathematics

Indian Institute of Technology Delhi

MTL 101 Linear Algebra and Differential Equations: Minor-I

Total marks: 20 Time: 1 hour

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1. Every question is compulsory
2. No marks will be provided for answers without proper justification

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1. Determine (with justification) whether the following statements are true or false. [$2 \times 4 = 8$]

(a) Let $V = \text{Maps}(\mathbb{N}, \mathbb{R})$, the space of all real sequences. Let e_n be the sequence $(a_m^{(n)})$, where $a_m^{(n)} = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n \end{cases}$, i.e., e_n is the sequence whose n^{th} term is 1 and all other terms are 0. Then $\{e_n | n \in \mathbb{N}\}$ forms a basis for V.

(b) Suppose $X_1, X_2 \in M_{n\times 1}(\mathbb{F})$ are solutions of the system $AX = B$, where $A \in M_{m\times n}(\mathbb{F})$ and $B \in M_{m\times 1}(\mathbb{F})$. Then, for $a, b \in \mathbb{F}$, $aX_1 + bX_2$ is a solution of $AX = B$ if $a + b = 1$.

(c) For $1 \leq i \leq 2n-1$, let $W_i = \{(x_1, x_2, \ldots, x_{2n}) | x_i + x_{i+1} = 0\} \subset \mathbb{R}^{2n}$ and let $W = \bigcap_{i=1}^{2n-1} W_i$. Then $\dim(W) = 0$.

(d) For any $n \in \mathbb{N}$ and $A, B \in M_{n\times n}(\mathbb{R})$, row space$(AB) \subseteq$ row space(A).

2. Let $X = \{1, 2, 3, 4, 5, 6, 7\}$ and let $V = \text{Maps}(X, \mathbb{R})$. Let $f_1, f_2, f_3, f_4 \in V$. The following table shows some of the values attained by the functions f_1, f_2, f_3 and f_4. (For e.g., $f_1(3) = -50$)

	1	2	3	4	5	6	7
f_1	100	1	-50	0	99	1	2
f_2	120	2	-21	2	-	1	3
f_3	-	2	71	1	30	1	2
f_4	0	1	-	0	50	1	3

Here - denotes unknown data. Is the set $\{f_1, f_2, f_3, f_4\}$ a linearly independent subset of V? [4]

3. Let B_1, B_2 and B_3 be any three distinct ordered bases of a three dimensional vector space V and let $B_3 = \{v_1, v_2, v_3\}$. Suppose that $[v_1]_{B_1} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $[v_2]_{B_1} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $[v_3]_{B_1} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $[v_1]_{B_2} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $[v_2]_{B_2} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $[v_3]_{B_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. Find a 3×3 matrix R such that for any $v \in V$, $[v]_{B_2} = R[v]_{B_1}$.

[4]

4. Find the inverse of the matrix

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 1 \end{pmatrix}$$

by reducing it to the identity matrix using elementary row operations.