

MTL101
LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
MINOR 1

Total Marks: 20

Time: One Hour

1. (4 Marks) Consider the linear system of equations in three unknowns (x, y, z) :

$$3x + 2y + az = 2,$$

$$9x + 2y + 3z = b,$$

$$6x + 8y + 5z = 5.$$

For what values of $a, b \in \mathbb{R}$, the system has:

- (a) No solution.
- (b) Unique solution.
- (c) Infinitely many solutions.

Also, when solution(s) exists, derive the expression for solution(s).

2. (4 Marks) Find the inverse of the following 3×3 matrix using Gauss-Jordan Method:

$$\begin{pmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 5 \end{pmatrix}.$$

3. (4 Marks) Let W_1 and W_2 be two subspaces of a vector space, then PROVE or DISPROVE the following statements:

- (a) $W_1 \cup W_2$ is a subspace.
- (b) $W_1 \cap W_2$ is a subspace.

4. (4 Marks) Consider \mathbb{P}_3 , the vector space of all polynomials of degree at most 3, over \mathbb{R} . Expand the set

$$S = \{x + x^2, x + x^2 + x^3\},$$

to form a basis of \mathbb{P}_3 .

5. (4 Marks) Prove that the set

$$S = \{e^x, \sin(x), \cos(x)\}$$

is Linearly Independent in vector space of continuous functions from \mathbb{R} to \mathbb{R} .