MINOR TEST 2 (MTL101)

Maximum Time: 1 Hour Max Marks: 20

All questions carry equal marks

1. Let V be the vector space of all polynomials of degree at most 1 over $\mathbb{R}$, the field of real numbers, w.r.t. the usual addition and scalar multiplication.

   Determine the dual basis $\hat{B} = \{\hat{u_1}, \hat{u_2}\}$ for the subspace U of V, spanned by the vectors $u_1 = 1 - 2x, u_2 = 3 + x$. Also, determine $\hat{u_2}(3 + x)$.

2. Let V be the vector space as given in question 1. Define for all $f \equiv f(x),\ g \equiv g(x) \in V$

$$\langle f, g \rangle = \frac{1}{2}\{f(0)g(0) + f(1)g(1)\}.$$

   Does it define an inner product on V? If yes, determine the norm of $h(x) = \frac{1}{2} - 2x \in V$.

3. Let V be a vector space of dimension n over a field $\mathbb{F}$, and W be its subspace of dimension r.

   Prove that $\hat{W}$ is isomorphic onto $\frac{\hat{V}}{A(W)}$. Hence or otherwise determine $dim_{\mathbb{F}} A(W)$.

   Here $A(W)$ denotes the annihilator of W over $\mathbb{F}$.

4. Find all solutions of the following system of equations over the field $\mathbb{Z}_5$ (integers modulo 5).

$$x_1 + 4x_4 = 0$$

$$x_1 + 2x_2 + 4x_3 = 0$$

$$2x_1 + 2x_2 + x_4 = 0$$

$$x_1 + 3x_3 = 0.$$