Minor Test 2 ::: MTL 101 ::: March 2016

Every question is compulsory.
Marks will not be awarded if appropriate arguments are not provided.
Do not write the questions. Write answers only.
Do not waste time describing what is not asked.

Maximum Marks: 20 Maximum Time: one hour

(1) Suppose $V = \mathbb{R}^3$ is provided the following inner product:

$$\langle (x_1, x_2, x_3) | (y_1, y_2, y_3) \rangle = x_1y_1 + 2x_2y_2 + 3x_3y_3 + x_1y_2 + x_2y_1 + x_3y_1 + x_1y_3.$$

Find the best approximation of $v = (1, 2, 3)$ by a vector of the XZ-plane (i.e., $\{(x, y, z) \in \mathbb{R}^3 : y = 0\}$). Also find the shortest distance of $(1, 2, 3)$ from the XZ-plane with respect to the given inner product. [4]

(2) Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by

$$T(x, y, z) = (7x + 2y + 3z, 8y, x - 2y + 5z).$$

Find a basis of $\mathbb{R}^3$ with respect to which the matrix of T is a diagonal matrix. [4]

(3) Find whether the following statements are true or false. Justify your answer. [$4 = 2 \times 2$]

(a) The following is an inner product on $\mathbb{R}^2$:

$$\langle (x_1, x_2) | (y_1, y_2) \rangle = x_1y_1 + 2x_1y_2 + 2x_2y_1 + x_2y_2.$$

(b) A linear operator $T : V \to V$ is not invertible if zero is an eigenvalue of T.

(4) Find the general solution of the following ODE: [4]

$$(3xy + 2y^2 + 4y)dx + (x^2 + 2xy + 2x)dy = 0.$$

(5) Consider the IVP: [$4 = 2 + 2$]

$$(x^2 - 1)y' = 4y, \; y(x_0) = y_0.$$

(a) Find the values of (x_0, y_0) for which a unique solution is guaranteed by the existence-uniqueness theorem.

(b) Show that if $(x_0, y_0) = (1, 0)$, then the IVP has infinitely many solutions.

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