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2017CS10328

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Indian Institute of Technology Delhi

MTL 101 Linear Algebra and Differential Equations: Minor-II

Total marks: 20 Time: 1 hour

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1. Every question is compulsory
2. No marks will be provided if appropriate justification is not provided

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1. (a) Find a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ such that the range space of T is spanned by $\{(1,0,1,0),(0,1,-2,1),(1,1,-1,1)\}$. [3]

   (b) Find the nullity of the transformation you have found. [1]

2. Give an example to justify the following statement: [2]

   "The sum of two diagonalizable matrices need not be diagonalizable".

3. Define $T : \mathbb{R}^4 \to \mathbb{R}^4$ as $T(x,y,z,w) = (x+z, y+z, 2z, 2w)$.

   (a) Find a basis B of $\mathbb{R}^4$ such that $[T]_B$ is a diagonal matrix. [4]

   (b) Find $P \in M_{4\times4}(\mathbb{R})$ such that $[T]_S = P[T]_B P^{-1}$, where S denotes the standard basis of $\mathbb{R}^4$. [1]

4. (a) Find as many solutions, as you can, of the IVP

$$y - \sin(x)\frac{dy}{dx} = 0,\ y(0) = 0. \qquad [2]$$

   (b) For what values of y_0 does the IVP

$$y - \tan(xy)\left(\frac{dy}{dx}\right)^2 = 0,\ y(0) = y_0$$

   possess **NO** solutions? [1]

5. Discuss existence and uniqueness of the IVP $y = g(x)\frac{dy}{dx}$, $y(0) = 1$, where

$$g(x) = \begin{cases} \dfrac{\sin x}{x} & ; \ x \neq 0 \\ 1 & ; \ x = 0. \end{cases} \qquad [3]$$

6. Find the first four Picard's approximation (upto y_4) of the solution of the IVP

$$y' = 2xy,\ y(0) = 1. \qquad [3]$$