MTL103: Major exam on May 2, 2015

Max Marks: 35 | **Time: 2 Hrs**

All questions are compulsory.

Q1. Let Q be an $n \times n$ symmetric matrix. Consider the following nonlinear program

$$(P) \qquad \min \frac{1}{2}x^tQx \qquad \text{subject to} \quad \frac{1}{2}x^Tx \leq \frac{1}{2}, \ x \in \mathbb{R}^n.$$

Show that (P) and its Lagrange dual have same optimal values.
(Hint: use idea of minimum eigen-value of Q) **[6]**

Q2. Consider the nonlinear programming problem

$$\begin{aligned} \min \ & 4x_1x_2 - x_1 - 2x_2 \\ \text{subject to} \ & \\ -x_1 + x_2 &\leq 0 \\ x_1 &\leq 1 \\ x_2 &\leq 1 \\ x_1, x_2 &\geq 0. \end{aligned}$$

(a) Find all KKT points of the problem.
(b) Does the constraint qualification met at any of these KKT points?
(c) What is the global solution to the problem? Justify your answer. **[6=3+2+1]**

Q3. Let A and B be an $m \times n$ and $m \times \ell$ matrices respectively. Prove, by using separation theorem of convex sets, that exactly one of the following two systems is consistent.

$$\begin{aligned} (I) \quad & Ax + By < 0, \ x \in \mathbb{R}^n, \ y \in \mathbb{R}^\ell \\ (II) \quad & \underset{=0}{A^Tp} + B^Tq = 0, \ p, q \geq 0, \ p, q \in \mathbb{R}^m \end{aligned}$$

[6]

Q4. Consider the following linear program

$$\begin{aligned} \min \ & -x_1 - 3x_2 \\ \text{subject to} \ & \\ x_1 + x_2 &\leq 6 \\ -x_1 + 2x_2 &\leq 6 \\ x_1, x_2 &\geq 0. \end{aligned}$$

It is desired to find an optimal solution and optimal basis when right hand side vector $b = (6,6)^T$ is perturbed along the direction $d = (-1,1)^T$, that is, b is changed to $b + \lambda d$, $\lambda \geq 0$. An optimal solution for $\lambda = 0$ is given below.

v_B	x_B	y_1	y_2	y_3	y_4	
x_1	2	1	0	2/3	-1/3	
x_2	4	0	1	1/3	1/3	
		0	0	-5/3	-2/3	$\leftarrow z_j - c_j$

(a) Find the range of λ for which the present basis remains optimal.
(b) Describe the optimal simplex table for the range of λ obtained in (a).
(c) Using the optimal table in (b), find an optimal solution when $\lambda = 2$.
(d) Is an optimal solution in (c) unique? If not, find an another alternate optimal solution. **[7=2+2+1+2]**

Q5. Find an optimal solution and optimal value of the dual to the following linear program

$$\begin{array}{rrcl} \max & 4x_1 + 6x_2 + 2x_3 + x_4 + 7x_5 + 5x_6 & & \\ \text{subject to} & & & \\ & x_1 + x_2 & \leq & 1 \\ & x_3 + x_4 & \leq & 1 \\ & x_5 + x_6 & \leq & 1 \\ & x_1 + x_3 + x_5 & = & 1 \\ & x_2 + x_4 + x_6 & = & 1 \\ & x_1, x_2, x_3, x_4, x_5, x_6 & \geq & 0. \end{array}$$

[5]

Q6. Consider the problem

$$\begin{array}{rrcl} \max & x_1 - 2x_2 + x_3 & & \\ \text{subject to} & & & \\ & x_1 + 2x_2 + 4x_3 - x_4 & \leq & 6 \\ & 2x_1 + 3x_2 - x_3 + x_4 & \leq & 12 \\ & x_1 + x_3 + x_4 & \leq & 4 \\ & x_1, x_2, x_3, x_4 & \geq & 0. \end{array}$$

Find a basic feasible solution with basis variables x_1, x_2, x_4. Can you suggest a better feasible solution ~~than this one~~? Why and how?

[5]