Department of Mathematics

MTL 103 (Optimization Methods and Applications)

Major Exam

Time: 2 hours
Max. Marks: 48

Date: 29/04/19

Note: The exam is closed-book, and all the questions are compulsory.

1. Consider a linear programming problem in standard form, i.e., $\min\{c^T x \mid Ax = b, x \geq 0\}$. Suppose that the matrix A has dimensions $m \times n$ and that its rows are linearly independent . For each one of the following statements, state whether it is true or false. If true, provide a proof, else, provide a counterexample.

   - If one problem has a non-degenerate and unique optimal basic feasible solution, so does the other.
   - Suppose that we have a non-degenerate optimal basis for the primal and that the reduced cost for one of the non-basic variables is zero. Then, exist another optimal basis.
   - The dual of the auxiliary primal problem considered in Phase I of the simplex method is always feasible.

   (6 marks)

2. Consider a linear programming problem

$$\min -x_1 - 2x_2 - 3x_3 + 2x_4$$
$$\text{s.t. } x_1 + 2x_2 + 3x_3 - 2x_4 = 15;\ 2x_1 + x_2 + 5x_3 - 3x_4 = 20;\ x_1 + 2x_2 + x_3 - x_4 = 10$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

   Consider a basis associated with basic variables x_1, x_2, x_3. a) Is it a optimal basis? If yes, find the associated optimal basic feasible solution and also comment on whether it is a unique optimal basic feasible solution, b) Does the LPP has infinite optimal solution? If yes, find all of them.

   (5 marks)

3. Let x^* be an optimal solution to the problem

$$\min c^T x$$
$$\text{s.t. } a_i^T x = b_i, \quad i = 1, 2, \cdots, m$$
$$x_j \geq 0, \ j = 1, 2, \cdots, n.$$

   Let w^* be an optimal dual solution. Show that x^* is also an optimal to the problem

$$\min (c^T - w_k^* a_k^T) x$$
$$\text{s.t. } a_i^T x = b_i, \quad i = 1, 2, \cdots, m, i \neq k$$
$$x_j \geq 0, \ j = 1, 2, \cdots, n,$$

   where w_k^* is the kth component of w^*.

   (7 marks)

4. Consider the following LPP problem and its optimal tableau:

$$\min \quad -2x_1 - x_2 + x_3$$
$$\text{s.t.} \quad x_1 + 2x_2 + x_3 + x_4 = 8; \quad -x_1 + x_2 - 2x_3 + x_5 = 4;$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

c_B	x_B	b	x_1	x_2	x_3	x_4	x_5
-2	x_1	8	1	2	1	1	0
0	x_5	12	0	3	-1	1	1
		16	0	3	3	2	0

(a) Find new optimal solution if the coefficient of x_2 in the objective function is changed from -1 to -6.

(b) Find new optimal solution if the coefficient of x_2 in the first constraint is changed from 2 to $\frac{1}{4}$.

(c) What will be the change in the optimal solution, if a new constraint $x_2 + x_3 = 3$ is added?

(d) What will be the change in the optimal solution, if a new variable x_6 is introduced with cost coefficient 4 and consumption vector A_6 as $(1,2)^T$?

(10 marks)

5. Consider the following integer programming problem

$$\min \ -x_1 - 4x_2$$
$$\text{s.t.} \ \ 2x_1 + 4x_2 + x_3 = 7; \ \ 5x_1 + 3x_2 + x_4 = 15,$$
$$x_1, x_2, x_3, x_4 \geq 0; \ x_1, x_2, x_3, x_4 \text{ integer}.$$

The optimal simplex tableau for LP relaxation problem is given by

c_B	x_B	b	x_1	x_2	x_3	x_4
-4	x_2	$\frac{7}{4}$	$\frac{1}{2}$	1	$\frac{1}{4}$	0
0	x_4	$\frac{39}{4}$	$\frac{7}{2}$	0	$\frac{-3}{4}$	1
		7	1	0	1	0

Using Gomory's fractional cut method find the optimal solution.

(7 marks)

6. Solve the following integer linear programming problem using branch and bound method

$$\min \ x_1 - 2x_2$$
$$\text{s.t.} \ \ -4x_1 + 6x_2 \leq 9; \ x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0; \ x_1, x_2 \text{ integer}$$

(6 marks)

7. The following table shows all the necessary information on the available supply to each warehouse, the requirement of each market and the unit transportation cost from each warehouse to each market

	I	II	III	IV	Supply
A	5	2	4	3	22
B	4	8	1	6	15
C	4	6	7	5	8
Demand	7	12	17	9	45

a) Find initial basic feasible solutions using North-West corner, least-cost and Vogel approximation methods.

b) Start with initial basic feasible solution obtained from least-cost method and find the optimal schedule and minimum total shipping cost.

(7 marks)