Major Exam: MTL103

May 01, 2017

Max Marks 45

Max Time 2 hrs

1. The following tableau represents the initial simplex table for a ~~minimization~~ maximization problem in standard form.

v	x	y_1	y_2	y_3	y_4	y_5	y_6	y_7
x_2	β	0	1	0	α	1	0	3
x_3	2	0	0	1	-2	2	η	-1
x_1	3	1	0	0	0	-1	2	1
	z	0	0	0	δ	3	γ	ξ

For each of the following statements, find the ranges of values of the various parameters, if they exist, that will make the given statement true.

Do all parts of this question at one place in answer script. You will be awarded (-4) marks for violating this instruction.

(i) Phase II of the simplex method can be applied using this as an initial tableau.

(ii) The present table indicates that the given problem is infeasible.

(iii) The corresponding basic solution is feasible, but we do not have an optimal basis in the present table.

(iv) The corresponding basic solution is feasible but the present table indicates that the given problem is unbounded.

(v) The corresponding basic solution is feasible, x_6 is a candidate for entering the basis and x_3 leaves the basis.

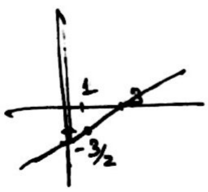
(vi) The corresponding basic solution is feasible, x_7 is a candidate for entering the basis, but if it does, then the solution and the objective value remain unchanged in the next table.

(vii) The current solution is non-degenerate, x_4 is a candidate for entering in the basis and the solution in next iteration is degenerate.

(viii) The current solution is degenerate, x_7 is a candidate for entering in the basis and the solution in next iteration is non-degenerate. [8]

2. Write the dual of the following problem:

$$\begin{aligned} \min \quad & |x_1| + |x_2| + |x_3| \\ \text{subject to} \quad & \\ & x_1 - 2x_2 = 3 \\ & -x_2 + x_3 \leqslant 1 \end{aligned}$$



Determine the optimal solution of the dual problem. Using the duality theory, find the optimal solution of the above problem. No marks will be awarded for using any other approach to solve the problem. [8]

3. Prove that the coefficient matrix in a transportation programming problem with m origins and n destinations is a unimodular matrix of rank $n + m - 1$. [8]

4. Consider the profit maximization assignment problem:

	J_1	J_2	J_3	J_4	J_5	J_6
P_1	6	12	3	11	15	13
P_2	8	11	10	7	11	10
P_3	9	7	6	10	12	9

Each person has to do minimum one job;
P_1 and P_3 agree to do at most 2 and 3 jobs respectively;
P_2 refused to do job 3 but agrees to help in completing all remaining jobs if assigned to him.
Find the optimal assignment and optimal profit. Original table
Write all steps very clearly and explaining your intend. [7]

5. Using the KKT conditions, find the optimal solution(s) of the following nonlinear programming problem and determine if the solution(s) is (are) global optimal solution(s)?

$$\begin{aligned} \max \quad & f(x,y) = xy \\ \text{subject to} \quad & x + y^2 \leqslant 2 \\ & x + \sqrt{2}\,y \geqslant 2 \\ & x, y \geqslant 0 \end{aligned}$$

~~Subtract~~
Negate. [7]

6. Does the BCQ hold at the point $(x_1, x_2) = (1, 14)$ for the following set of inequalities? Justify.

$$-x_1^3 + 6x_1^2 - 9x_1 + x_2 - 10 \leqslant 0, \quad -x_2 + 14 \leqslant 0, \quad x_1 - 5 \leqslant 0.$$

[7]