$x_3 - \frac{x_5}{6} + x_6 + \frac{5x_7}{6} = 5 + 5/6$

$x_3 = 5 + \frac{5}{6} - \left(x_6 + \frac{5x_7}{6} - \frac{x_5}{6} \right)$

Department of Mathematics

MTL 103 (Optimization Methods and Applications)

Major Exam

Time: 2 hours
Max. Marks: 48

Date: 07/05/18

Note: The exam is closed-book, and all the questions are compulsory.

Q.1 Consider the following LP problem

$$\min\ -3x_1 - 2x_2 - 5x_3$$
$$\text{s.t. } x_1 + 2x_2 + x_3 \le a_1,\quad 3x_1 + 2x_3 \le a_2,\quad x_1 + 4x_2 \le a_3$$
$$x_1, x_2, x_3 \ge 0,$$

where a_1, a_2, a_3 are constants. Let x_4, x_5, x_6 be slack variables. For specific values of a_1, a_2 and a_3 the optimal tableau is

x_B	b	x_1	x_2	x_3	x_4	x_5	x_6
x_2	100	b_1	1	0	$\frac{1}{2}$	$\frac{-1}{4}$	0
x_3	c_3	b_2	0	1	0	$\frac{1}{2}$	0
x_6	20	b_3	0	0	-2	1	1
	-1350	4	0	0	c_1	c_2	0

where b_i's and c_i's are constants. Determine
(*i*) The values of a_1, a_2 and a_3 that yield the given optimal solution.
(*ii*) The values of b_1, b_2, b_3 and c_1, c_2, c_3 in the optimal tableau.

(8 Marks)

Q.2 Consider a linear programming problem in standard form which is infeasible, but which becomes feasible and has finite optimal cost when the last equality constraint is omitted. Show that the dual of the original (infeasible) problem is feasible and the optimal value is infinite.

(4 Marks)

Q.3 Consider a linear programming problem in standard form and its dual. Then,

(a) Prove that if one problem has a non-degenerate and unique optimal basic feasible solution, so does the other.

(b) Suppose that we have a non-degenerate optimal basis for the primal and that the reduced cost for one of the non-basic variables is zero. Is it true that there must exist another optimal basis? Support your claim with proper justifications.

(5 Marks)

Q.4 Consider the following mixed integer linear programming problem

$$\min -4x_1 - 6x_2 - 2x_3$$
$$\text{s.t. } 4x_1 - 4x_2 + x_4 = 5,\quad -x_1 + 6x_2 + x_5 = 5,\quad -x_1 + x_2 + x_3 + x_6 = 5$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0; x_1 \text{ and } x_3 \text{ are integers.}$$

The optimal tableau for LP relaxation problem is given below:

c_B	x_B	b	x_1	x_2	x_3	x_4	x_5	x_6
-4	x_1	$\frac{5}{2}$	1	0	0	$\frac{3}{10}$	$\frac{1}{5}$	0
-6	x_2	$\frac{5}{4}$	0	1	0	$\frac{1}{20}$	$\frac{1}{5}$	0
-2	x_3	$\frac{25}{4}$	0	0	1	$\frac{1}{4}$	0	1
		-30	0	0	0	2	2	2

Using mixed integer cut find the optimal solution of mixed integer linear programming problem.

(6 Marks)

Q.5 Solve the following integer linear programming problem by applying fractional cuts to LP relaxation problem

$$\max\ 7x_1 + 10x_2$$
$$\text{s.t}\quad -x_1 + 3x_2 \le 6,\quad 7x_1 + x_2 \le 35,$$
$$x_1, x_2 \ge 0 \text{ and integer.}$$

(7 Marks)

Q.6 A company has factories at F_1, F_2 and F_3 which supply warehouses at W_1, W_2 and W_3. The unit shipping costs, factories capacities and warehouses requirements are summarized in the table below:

	W_1	W_2	W_3	*Supply*
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	16	90
Demand	180	120	150	

Find initial feasible solution using Vogel's approximation method and determine the optimum distribution for this company to minimize shipping costs.

(6 Marks)

Q.7 The optimal simplex tableau of a LP in minimization form is given by

x_B	b	x_1	x_2	x_3	x_4
x_2	6	0	1	2	-1
x_1	2	1	0	-1	1
		0	0	2	1

Assume that x_3 and x_4 are slack variables and the starting basis was $(x_3, x_4)^T$. Then,

(i) Write the original LP.

(ii) Let a new variable x_5 is introduced with $c_5 = -15$ and $A_5 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. What will be the new optimal solution.

(6 Marks)

Q.8 Consider an optimization problem

$$\min \sum_{i=1}^{n} c_i |x_i|$$
$$\text{s.t.}\quad Ax \le b$$

where $c_i \ge 0$, for all $i = 1, 2, \cdots, n$. Formulate this problem as an equivalent linear programming problem. Justify your claims with proper arguments.

(6 Marks)