MTL103: Major Exam on 15/05/2021 starting at 9:30 am

Max Marks 40 Max Time: 135 minutes till 11:45 am including the uploading time

The paper has six questions. Start answering each question on a new page.
Write your name and entry number on top of each page.
For each question, do all parts of the question at one go, clearly mentioning the part number in your answer - no recheck/regrade request will be entertained on mixing the parts and/or not correctly mentioning the part number in the answer.
Upload your PDF file on Gradescope ONLY with clear and correct mapping of answers.
Submission of answer sheet on any other platform or emailing will NOT be considered for any reason.
Copying even one question or part of it will amount to cancellation of the major exam plus -10 marks penalty

No marks will be awarded if answer is not supported with correct justification and working.

1. A maximization LPP has three original variables $x_1,\ x_2,\ x_3$ and two slack variables s_1 and s_2 corresponding to the inequality constraints. All five variables are non-negative. The following tableau represents a basic feasible solution of such an LPP.

x_B	y_1	y_2	y_3	y_{s_1}	y_{s_2}
4	0	-5/4	1	1/2	1/4
2/3	1	1/2	0	-1/3	1/6
$z_j - c_j \rightarrow$	0	-10	0	3	4

Construct the original LPP. Find the optimal solution of this LPP, if it exists? Can we achieve the objective value equals 136 for this LPP? If so, how? [7]

2. Consider the linear program (P):
max $\quad z = 12x_1 + 5x_2 + 7x_3$
subject to

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &\leq 420 \\ 3x_1 + x_2 - 5x_3 &\leq 320 \\ x_1,\ x_2,\ x_3 &\geq 0. \end{aligned}$$

Let (D) be its dual. Without solving (P) directly at any stage, answer the following two parts:

(i) If $(2,\ \alpha,\ 4)$ is a feasible solution of (P) and $(\beta,\ 1/2)$ is a feasible solution of (D) then show that the optimal value of (P) lies in the interval [1730, 2365].

(ii) Use the complementary slackness theorem to find the optimal solution and hence optimal value of (P). [7]

3. A binary variable can be used to represent the Boolean expression in propositional logic. For instance, if Q is a logical statement that can take values True or False then we can attach a binary variable y with Q which takes the value 1 or 0 corresponding to True or False values of Q, respectively. So, a binary variable $1 - y$ gets attached with $\neg Q$ (negation of Q). Thus, the logical disjunctive $Q_1 \vee Q_2$ is equivalent to $y_1 + y_2 \geq 1$. Use this information to write the mathematical linear inequalities representing each of the following Boolean expressions attaching the binary variables y_i with Q_i, $i = 1, 2, 3, 4$:

(i) $(Q_1 \vee Q_2) \Rightarrow (Q_3 \vee Q_4)$
(ii) $(Q_1 \Rightarrow (Q_2 \vee Q_3)) \wedge (Q_4 \Rightarrow (\neg Q_2 \wedge Q_1))$
(iii) If Q_1 is True and Q_2 is False then Q_3 is True
(iv) If exactly one of the Q_1 is True or Q_2 is True then Q_3 is False. [8]

4. Use the branch and bound method to solve the following integer linear program:
max $\quad z = 2x_1 + x_2$
subject to

$$\begin{aligned} x_1 + x_2 &\leq 5 \\ -x_1 + x_2 &\leq 0 \\ 6x_1 + 2x_2 &\leq 21 \\ x_1,\ x_2 &\geq 0 \text{ and both integers.} \end{aligned}$$

You can use graphical method to solve optimization model at each node of the branch and bound tree. Choose the variable with the largest fraction for branching at the node.

Provide the tree diagram depicting the LPP solved and the action taken at each node of the tree. [6]

5. The SPIC MACAY society of our institute decided to hold online instrument program on each – Tabla, Sitar, Santoor, and Flute. The scheduling of these programs has to be done so that the total audience attending these programs is kept at maximum. Only one program can be held in a day and there must be one program of each of the four instruments organized in a week.

   The artists of the programs provide additional information:- Santoor can not be held on Tuesday; Flute can not be held on Friday; a second program of only one of either Tabla or Flute can be arranged in the same week on some other day.

   From the past experience, a matrix of number of attendees who are expected to attend a particular program on a specific day is given as follows:

	Tabla	Sitar	Santoor	Flute
Monday	50	40	60	20
Tuesday	40	30	40	30
Wednesday	50	20	30	20
Thursday	40	50	30	50
Friday	30	30	20	30
Saturday	60	30	50	40

   Find the optimal program schedule and maximum expected audience attending these programs. Show the steps of the procedure very clearly in readable form. [6]

6. Let $\{(1,1), (1,4), (2,3), (2,4), (3,2), (3,4)\}$ be the set of basic cells in the following cost minimization transportation problem. Allocate the values to these cells and find the BFS. Using this BFS, find the optimal transportation allocations from origins to destinations. Show the steps very clearly in readable form. [6]

	D_1	D_2	D_3	D_4	a_i
O_1	19	30	50	12	7
O_2	70	30	40	60	10
O_3	40	10	60	20	18
b_j	5	8	7	15	35