DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
MINOR-II 2015-2016 SECOND SEMESTER
MTL103/MAL210 (OPTIMIZATION METHODS AND APPLICATION)

Time: 1 hour Max. Marks: 25

** Answer to each question should begin on a new page **

1. Let $x^* = [x_1^*, x_2^*, \ldots\ldots, x_n^*]^T$ be a feasible solution of the linear programming problem (P) and $y^* = [y_1^*, y_2^*, \ldots\ldots, y_m^*]^T$ be a feasible solution of the dual of the linear programming problem (D). Then prove x^* is an optimal solution of (P) and y^* is an optimal solution of (D) simultaneously if and only if both of the following statements hold

$$x_j^* = 0 \text{ or } \sum_{i=1}^m a_{ij} y_i^* = c_j \text{ for all } j = 1, 2, \ldots\ldots, n$$
$$y_i^* = 0 \text{ or } \sum_{j=1}^n a_{ij} x_j^* = b_i \text{ for all } i = 1, 2, \ldots\ldots, m. \quad (3)$$

2. Use the dual Simplex method to solve the linear programming problem

$$\text{Minimize: } Z = 10x_1 + 6x_2 + 2x_3$$
$$\text{subject to: } -x_1 + x_2 + x_3 \geq 1$$
$$3x_1 + x_2 - x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0. \quad (4)$$

3. Consider the quadratic programming (QP) problem

$$\text{Minimize: } \tfrac{1}{2} x^T Q x + c^T x$$
$$\text{subject to: } Ax \leq b,$$

where Q is an $n \times n$ symmetric positive definite matrix, $c \in R^n$, $A \in R^{m \times n}$, $b \in R^n$. Then prove or disprove: the dual of a convex QP problem is a concave QP problem. (5)

4. Consider the convex problem with equality constraints.

$$\text{Minimize: } f(x)$$
$$\text{subject to: } h(x) = 0$$

where $f(x)$ is a convex function and $X = \{x \in R^n : h(x) = 0\}$ is a convex set. Let x^* be a regular point satisfying Lagranges theorem,

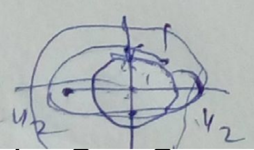
$$h(x^*) = 0, \text{ and}$$
$$\nabla f(x^*) + \sum_{i=1}^m \lambda_i \nabla h_i(x^*) = 0.$$

Then prove or disprove that x^* is a global minimizer. (4)

5. Consider the problem of optimizing $f(x) = x^T Q x$ subject to a single equality constraint $x^T P x = 1$, where Q is a symmetric positive semi-definite matrix and P is a symmetric positive definite matrix. Then establish that an eigenvector corresponding to the smallest (largest) eigenvalue of $P^{-1}Q$ is a global minimizer (maximizer) of this problem. (4)

6. Find local minimizer(s) of the problem:

$$\text{Minimize: } x_1^2 + x_2^2 \quad = r^2$$
$$\text{subject to: } 4x_1^2 + x_2^2 - 1 = 0$$



(5)