Department of Mathematics

MTL 103 (Optimization Methods and Applications)

Minor Exam 2

Date: 27/03/18

Time: 1 hour
Max. Marks: 20

Note: The exam is closed-book, and all the questions are compulsory.

Q.1 (a) Consider a linear program (LP) in standard form

$$\min \ -107x_1 - x_2 - 2x_3$$

s.t.

$$14x_1 + x_2 - 6x_3 + 3x_4 = 7$$
$$16x_1 + x_2 - 6x_3 + x_5 = 5$$
$$3x_1 - x_2 - x_3 + x_6 = 0$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0.$$

Take a basic feasible solution $x = (0, 0, 0, \frac{7}{3}, 5, 0)$ and check for the unboundedness of the LP. If LP is unbounded, justify your claim by finding a direction along which cost function can be reduced arbitrarily. Also support your claim with proper arguments in case LP is not unbounded.

(3 marks).

(b) Consider a LP in standard form

$$\min \ -5x_1 + 4x_2 - 3x_3$$

s.t.

$$2x_1 + x_2 - 6x_3 = 20$$
$$6x_1 + 5x_2 + 10x_3 + x_4 = 76$$
$$8x_1 - 3x_2 + 6x_3 + x_5 = 50$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0.$$

$C_B = \begin{pmatrix} -5 \\ 4 \\ 0 \end{pmatrix}$

$\begin{bmatrix} 2 & 1 & -6 \\ 6 & 5 & 10 \\ 8 & -3 & 6 \end{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Without using simplex tableau method show that the basis corresponding to basic variables x_1, x_2 and x_4 is optimal. Obtain the optimal solutions of both primal and dual problem.

(3 marks)

Q.2 Solve the following LP using Big-M method

$$\max \ 6x_1 + 4x_2$$

s.t.

$$2x_1 + 3x_2 \leq 30$$
$$3x_1 + 2x_2 \leq 24$$
$$x_1 + x_2 \geq 3$$
$$x_1 \geq 0, x_2 \geq 0.$$

Is the optimal solution unique? If not, give two different optimal solutions.

(6 marks)

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$$\bar{c}^T\bar{x} \le \bar{c}^T x^*$$
$$c^T x^* \le c^T\bar{x}$$

Q.3 Consider the following LP in standard form

$$\min\ c^T x$$
s.t.
$$Ax = b$$
$$x \ge 0.$$

Let x^* be an optimal solution, assumed to exist , and let p^* be an optimal solution to the dual.

(a) Let $\bar{x}$ be an optimal solution to the primal, when c is replaced by some $\bar{c}$. Show that

$$(\bar{c} - c)^T(\bar{x} - x^*) \le 0.$$

(b) Let the cost vector be fixed at c, but suppose that we now change b to $\bar{b}$, and let $\bar{x}$ be a corresponding optimal solution to the primal. Prove that

$$(p^*)^T(\bar{b} - b) \le c^T(\bar{x} - x^*).$$

(3 marks)

Q.4 Consider the following primal LP

$$\min \quad 2x_1 + 15x_2 + 5x_3 + 6x_4$$
s.t
$$x_1 + 6x_2 + 3x_3 + x_4 \ge 2 \quad p_1$$
$$2x_1 - 5x_2 + x_3 - 3x_4 \ge 3 \quad p_2$$
$$x_1, x_2, x_3, x_4 \ge 0.$$

Write the dual of of above LP. Then, without using any simplex tableau method find the solution of both primal and dual problem.

(5 marks)

$$p^{*T} b \ge p^{*T}\bar{b}$$
$$\cancel{c^T x^* \le c^T \bar{x}}$$
$$c^T x = p^{*T} b$$
$$c^T x^* \ge p^{*T}\bar{b} \qquad \ge p^{*T}\bar{b}$$
$$c^T x \ge p^{*T} b$$

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