DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

MINOR TEST I 2016-2017 FIRST SEMESTER

MTL104 (LINEAR ALGEBRA AND APPLICATIONS)

**Time: 1 hour** **Max. Marks: 25**

---

**1a.** If $W$ is any subspace of a vector space $V(F)$, then show that the set $\frac{V}{W}$ of all cosets $W+x$ where $x$ is any vector in $V(F)$ forms a vector space over $F$, under the operations defined by

$$(W+x) + (W+y) = W + (x+y), \quad x, y \in V$$

$$\alpha(W+x) = W + \alpha x, \quad \alpha \in F.$$

Also, prove that

$$\dim(\frac{V}{W}) = \dim V - \dim W.$$

(5)

**1b.** Let V(F) be a vector space. Let $W_1, W_2, \ldots\ldots, W_n$ be subspaces of V. Suppose

$$V = W_1 + W_2 + \ldots\ldots + W_n \quad \text{and} \quad W_i \cap \{\sum_{j=1,\ j\neq i}^{n} W_j\} = \{0\}, \quad 1 \le i \le n.$$

Prove or disprove that $V = W_1 \oplus W_2 \oplus \ldots\ldots \oplus W_n$. (3)

**2a.** Let $V$ and $W$ be two finite dimensional vector spaces and $N \subseteq V$, $R \subseteq W$ be two subspaces such that dim $N +$ dim $R =$ dim $V$. Is there a linear transformation $T \in L(V, W)$ such that $N(T) = N$ and $R(T) = R$? Give reasons for your answer. (5)

**2b.** Let $V(F)$ be the vector space of all arithmetic sequences over the field $F$ (real). i.e. all sequences of the form $\{a, a+d, a+2d, \ldots\ldots\}$. Then prove or disprove that $V(F)$ is isomorphic to $F^2$. (3)

**3.** Let p,m and n be positive integers and F be a field. Let V be the space of $m \times n$ matrices over F and W the space of $p \times n$ matrices over F. Let B be a fixed $p \times m$ matrix and let T be the linear transformation from V into W defined by $T(A) = BA$. Prove that T is invertible if and only if $p = m$ and B is an invertible $m \times m$ matrix. (3)

**4a.** Let $R^{2\times 2}$ denote the collection of all $2\times 2$ matrices with real elements. Define $f : R^{2\times 2} \longrightarrow R$, a linear functional as follows:

$$f\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = 2, \quad f\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = 3, \quad f\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = 4, \quad \text{and} \quad f\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = 9.$$

Determine a basis for $N(f)$, where $N(f)$ denotes null space of f. (3)

**4b.** Let $W_1$ and $W_2$ be subspaces of a finite dimensional vector space of V(F). Then prove or disprove that

$$(W_1 + W_2)^0 = W_1^0 \cap W_2^0.$$

(3)