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DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI
MINOR TEST 2022-2023 FIRST SEMESTER
MTL104 (LINEAR ALGEBRA AND APPLICATIONS)

Time: 1 hour **Max. Marks: 30**

**** Answer to each question should begin on a new page ****

1a. Let A be an $n \times n$ diagonal matrix with characteristic polynomial $(x-\lambda_1)^{d_1} \ldots\ldots, (x-\lambda_k)^{d_k}$, where $\lambda_1, \ldots\ldots, \lambda_k$ are distinct. Let V be the vector space of $n \times n$ matrices B such that $AB = BA$. Prove or disprove that the dimension of V is $d_1^2 + \ldots\ldots + d_k^2$. (4)

1b. Let $V = M_{2\times2}(R)$ be a vector space of all 2×2 matrices with the field R. Let W_1 and W_2 be subspaces of V defined by

$$W_1 = \left\{ \begin{pmatrix} a & b \\ c & a \end{pmatrix} \in V \ : \ a, b, c \in F \right\}$$

$\begin{bmatrix} 0 & -9 \\ 9 & 0 \end{bmatrix}$

and

$$W_2 = \left\{ \begin{pmatrix} 0 & a \\ -a & b \end{pmatrix} \in V \ : \ a, b \in F \right\}.$$

Find the dimension of the subspace $W_1 + W_2$. (3)

2a. Let p, m and n be positive integers and F be a field. Let V be the space of $m \times n$ matrices over F and W the space of $p \times n$ matrices over F. Let B be a fixed $p \times m$ matrix and let T be the linear transformation from V into W defined by $T(A) = BA$. Prove that T is invertible if and only if $p = m$ and B is an invertible $m \times m$ matrix. (4)

2b. Let V and U be vector spaces over the field F, and let $T : V \to U$ be a linear transformation. Then prove or disprove

$$\frac{V}{\text{Ker}(T)} \cong \text{Range}(T).$$

(4)

3a. Let F be a subfield of complex numbers. We define n linear functionals on $F^n (n \geq 2)$ by

$$f_k(x_1, x_2, \ldots\ldots, x_n) = \sum_{j=1}^{n} (k-j)x_j, \quad 1 \leq k \leq n.$$

Find dimension of the subspace annihilated by $f_1, f_2, \ldots\ldots, f_n$. (3)

3b. Let V be a vector space over the field F. Let V^* be dual space of V. Let $x_0 \in V$ and $f_0 \in V^*$. Define a linear operator $T \neq 0$ on V such that $T(x) = f_0(x)x_0$, for all $x \in V$. Then

(i) Determine T^t, i.e. give a rule for calculating $T^t g, \ g \in V^*$. (1)

(ii) Determine the $N(T^t)$. (2)

(iii) Determine a basis for $R(T^t)$. (2)

P.T.O.

4a. Let T be any linear operator on a finite dimensional vector space V. Let $\lambda_1, \lambda_2, \ldots\ldots, \lambda_k$ be the characteristic values of T, and let W_i be the space of characteristic vectors associated with characteristic value λ_i. Prove that $W_1, W_2, \ldots\ldots, W_k$ are independent. (3)

~~**4b.** If E_1 and E_2 are projections onto independent subspaces, then $E_1 + E_2$ is a projection.~~ **True or False?** Justify your answer. (2)

4c. If E is a projection and f is a polynomial, then $f(E) = aI + bE$. What are a and b in terms of the coefficients of f? (2)

$\boxed{\frac{m(m-1)}{2}}$

$\left(\frac{m(m-1)}{2}\right)$

$\frac{3x}{3}$

$\frac{2x(3)}{x}$ 6

$|A + A^T| = 0$

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