(T) $3 \cdot 4$

MTL 104, Minor 2

Indian Institute of Technology Delhi

Time: 1 Hour Max. Marks: 25

Attempt all questions. All notations are standard. All questions carry equal marks.

1. If V is an $n-$dimensional vector space over a field F and if $T \in A_{\mathbb{F}}(V)$ has all of its characteristic roots in $\mathbb{F}$, then prove that T satisfies a polynomial of degree atmost n over $\mathbb{F}$.

2. In each of the following cases, check if $T \in A_{\mathbb{F}}(V)$ is triangularizable, if yes determine its canonical form, also determine a regular $S \in A_{\mathbb{F}}(V)$ such that STS^{-1} is triangular.

   (i) $V = \mathbb{R}^2_{\mathbb{R}}, W = \mathbb{R}^2, \mathbb{F} = \mathbb{R}, (x, y)T = (x + y, x)$

   (ii) $V = \mathbb{R}^2_{\mathbb{R}}, W = \mathbb{R}^2, \mathbb{F} = \mathbb{R}, (x, y)T = (y, x)$

3. Let $T : \mathbb{R}^3_{\mathbb{R}} \to \mathbb{R}^3_{\mathbb{R}}$ be a linear transformation defined by $(x_1, x_2, x_3)T = (-x_2, x_1, x_3)$ and let $B = \{(1,0,0),(0,1,0),(0,0,1)\}$, $B' = \{(1,1,1),(1,-1,0),(0,0,1)\}$ be two ordered bases for $\mathbb{R}^3_{\mathbb{R}}$. Find a matrix P such that $[T]_{B'} = P[T]_B P^{-1}$.

4. Let V be the vector space of all polynomials in x over F of degree ≤ 5. Let $T : V \to V$ be defined by $(1)T = x^2 + x^4, (x)T = x + 1, (x^2)T = 1, (x^3)T = x^3 + x^2 + 1, (x^4)T = x^4, (x^5)T = 0$. If W is the linear span of $\{1, x^2, x^4\}$

   (i) Show that W is invariant under T.

   (ii) Find the matrix of T in a suitable basis of V.

   (iii) Find the matrix of $\overline{T}$ in a suitable basis of $\overline{V} = \frac{V}{W}$, where $\overline{T} : \overline{V} \to \overline{V}$ defined by $(\bar{v})\bar{T} = \overline{(v)T}$

5. Let $T : \mathbb{R}^3_{\mathbb{R}} \to \mathbb{R}^3_{\mathbb{R}}$ be a linear transformation defined as $(x, y, z)T = (3x + 2y + 2z, x + 2y + 2z, -x - y)$. Find the minimal polynomial of T, and also decompose $\mathbb{R}^3_{\mathbb{R}}$ as a direct sum of invariant subspaces.