DEPARTMENT OF MATHEMATICS
MTL 105: Algebra

Minor - 1 *Marks - 20*

(1) (a) In a group G if for some $a \in G$, $O(a) = |a| = n$ and k divides n. Prove that $|a^{\frac{n}{k}}| = k$. [2 marks]

(b) Find the inverse element of $\begin{pmatrix} 2 & 6 \\ 3 & 5 \end{pmatrix}$ in $GL_2(\mathbb{Z}_{11})$. [3 marks]

(2) Prove that for a group G, the *centralizer* of $a \in G$, $C(a)$, is a subgroup of G. Explain the relation between the *centre of* G, $Z(G)$, and $C(a)$ for each $a \in G$. [5 marks]

(3) Let $\mathbb{Z}$ denote the group of integers under addition. Is every subgroup of $\mathbb{Z}$ cyclic? Why? Describe all the subgroups of $\mathbb{Z}$. [5 marks]

(4) Prove that A_n (Alternating group of degree n) is a subgroup of S_n (Symmetric group of degree n). Explain why the order of A_n is $n!/2$. Discuss if D_4 is a subgroup of A_4. [5 marks]

(abc) (bca)

(acb) (cba)