

# DEPARTMENT OF MATHEMATICS

## MTL 105: Algebra

Minor - 1

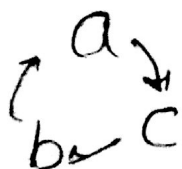
Marks - 20

- (1) (a) In a group  $G$  if for some  $a \in G$ ,  $O(a) = |a| = n$  and  $k$  divides  $n$ . Prove that  $|a^{\frac{n}{k}}| = k$ . [2 marks]
- (b) Find the inverse element of  $\begin{pmatrix} 2 & 6 \\ 3 & 5 \end{pmatrix}$  in  $GL_2(\mathbb{Z}_{11})$ . [3 marks]
- (2) Prove that for a group  $G$ , the *centralizer* of  $a \in G$ ,  $C(a)$ , is a subgroup of  $G$ . Explain the relation between the *centre* of  $G$ ,  $Z(G)$ , and  $C(a)$  for each  $a \in G$ . [5 marks]
- (3) Let  $\mathbb{Z}$  denote the group of integers under addition. Is every subgroup of  $\mathbb{Z}$  cyclic? Why? Describe all the subgroups of  $\mathbb{Z}$ . [5 marks]
- (4) Prove that  $A_n$  (Alternating group of degree  $n$ ) is a subgroup of  $S_n$  (Symmetric group of degree  $n$ ). Explain why the order of  $A_n$  is  $n!/2$ . Discuss if  $D_4$  is a subgroup of  $A_4$ . [5 marks]



$(abc)$      $(bca)$

$(acb)$      $(cba)$



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