DEPARTMENT OF MATHEMATICS
MTL 105: Algebra

Minor - 1 *Marks - 30*

[Upload your answers on Moodle, each answer separately.]

(1) State whether the following is **TRUE** or **FALSE**. Justify your assertion briefly.

[Marks are only for proper justification.]

a) In a group G if for some $a \in G$, $o(a) = |a| = n$ and k divides n then $o(a^{\frac{n}{k}}) = |a^{\frac{n}{k}}| = k$.

b) D_4 is a subgroup of A_4.

c) Let H, K be subgroups of a finite group G such that $H \leq K \leq G$, then $i_G(H) = i_G(K)i_K(H)$.

d) There are 6 distinct abelian groups (upto isomorphism) of order 360.

e) The last two digits of 31^{321} is 31. [10 marks]

(2) Prove that a cyclic group with only one generator can have atmost 2 elements. [5 marks]

(3) For $n \geq 2$ show that the smallest subgroup of S_n containing the cycles $(1,2)$ and $(1,2,\ldots,n)$ is S_n. [5 marks]

(4) For a subgroup H of a group G define $aH * bH = abH, \ \ \forall a, b \in G$. Show that all the left cosets of the subgroup $H = \{0\} \oplus \mathbb{Z}_2$ of the group $G = \mathbb{Z} \oplus \mathbb{Z}_2$ forms a group with the operation $*$ as defined and this group of all left cosets of H is isomorphic to the group $\mathbb{Z}$. [5 marks]

(5) (a) Let $G = \mathbb{Z}_4 \oplus \mathbb{Z}_4$. Determine if the subgroups $H = \{(0,0),(2,0),(0,2),(2,2)\}$ and $K = \langle (1,2) \rangle$ of G are isomorphic. Justify your answer.

(b) If groups G and G' are isomorphic and G is abelian, then prove that G' is abelian. [3+2 marks]