20th September 2021

**MTL105**

MINOR EXAMINATION

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You shall stop writing at 10:30 AM. Then you have 15 minutes to scan and upload your answer script on Gradescope. You shall NOT scan your answer script before 10:30 AM. After 10:45 AM answer scripts will not be accepted.

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(1) Let $\alpha = (1\ 3)(4\ 5)$ and $\beta = (1\ 2\ 4\ 5)$ in $S_5$. Find $\gamma \in S_5$ such that $\alpha\gamma = \beta$. 2 marks

(2) Let $G$ be a group of order 143. Show that there exists subgroups $H, K$ of $G$ such that $G = HK$. 2 marks

(3) Let $G$ be a group and $H$ be a non-empty finite subset of $G$. Show that $H$ is a subgroup of $G$ if and only if for all $a, b \in H$, $ab \in H$. 3 marks

(4) Find all the normal subgroups of $S_4$. 3 marks

(5) Find out all the group homomorphisms from the group $S_3$ to the group of non-zero complex numbers $\mathbb{C}^*$. 4 marks

(6) Show that for $n \geq 3$, the cycle $(1\ 2\ 3)$ is not a cube of any element in $S_n$. 4 marks

(7) Let $G$ be a group of order 231. Prove that the 11 - Sylow subgroup of $G$ is in the center. 4 marks

(8) Let $G$ be a group of order 385. Show that its 11-Sylow subgroup is normal and its 7-Sylow subgroup is in the center of $G$. 4 marks

(9) Let $G$ be a group of order $p^n$, where $p$ is a prime and $n \in \mathbb{N}$. Show that any subgroup of $G$ of order $p^{n-1}$ is a normal subgroup. 4 marks

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