2022|28K

4 8 9 2

min 28K ≡ 0(mod 20
K

MINOR EXAMINATION (MTL105)
ACADEMIC SESSION: 2022-23
TOTAL MARKS: 35
DURATION: 1 HOUR 20 MINUTES

---

(1) Find out the order of [28] in the group $\mathbb{Z}/2022\mathbb{Z}$. **2 marks**

(2) Let $\sigma = (1\ 2\ 3\ \cdots\ 12)$. Find out all the integers $1 \le k \le 12$ such that σ^k is also a 12 - cycle? **2 marks**

(3) Let G be a group and H a subgroup of prime order. Prove that for any subgroup K of G either $H \cap K = \{e\}$ or $H \subseteq K$. **2 marks**

(4) State True/False with justification: **3+2+2+2+2=11 marks**

a) Any subgroup in a cyclic group is a characteristic subgroup.

b) There is no infinite non-abelian group having every element of finite order.

c) Any group having only two subgroups is a cyclic group.

d) The groups $\mathbb{Q}$ and $\mathbb{Q} \times \mathbb{Z}/3\mathbb{Z}$ are isomorphic.

e) There is a non-cyclic group of order 289.

(5) Let G be a finite group. Suppose there exists an element $a \in G$ such that the conjugacy class of a contains two elements. Prove that G has a non-trivial normal subgroup. **3 marks**

(6) Let G be a group acting on a set $A \neq \varnothing$. Let $a \in A$ and $b \in \mathrm{Orb}(a)$.

a) Prove that $G_b = gG_ag^{-1}$ for some $g \in G$.

b) Prove that if $A = \mathrm{Orb}(a)$, then the kernel of the group action is $\bigcap_{g \in G} gG_ag^{-1}$.

2+2=4 marks

(7) Up to isomorphism find out the automorphism group of the group $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$. **3 marks**

(8) Let G be a group such that the group of inner automorphisms of G is cyclic. Prove that G is abelian. **3 marks**

(9) Let G be a group of order 231. Prove that there is only one 11 - Sylow subgroup and it is contained in the center. **2+3 marks**

Date: September 28, 2022.