Minor Test-II

Course: MTL 105 **Duration: 1 hour** **M. Marks: 25**

Note: All questions are compulsory.

I. Prove that if a group G of order 28 has a normal subgroup of order 4, then G must be Abelian. **[5 marks]**

II. Prove that two finite Abelian groups are isomorphic if and only if they have the same set of invariants. **[5 marks]**

III. Let G be a finite group in which the number of solutions in G of the equation $x^n = e$ is at most n for each positive integer n. Prove that G is a cyclic group. **[5 marks]**

IV. Prove that the dihedral group D_6 of order 12 is isomorphic to $S_3 \times \mathbb{Z}/2\mathbb{Z}$. **[5 marks]**

V. Let R be a ring with more than one element. Suppose that for each $a \in R$, there exists a unique $b \in R$ such that $aba = a$. Prove that $bab = b$ and R is a division ring. **[5 marks]**