DEPARTMENT OF MATHEMATICS
MTL 105: Algebra

Minor - 2 *Marks - 20*

[You may assume everything done in class.]

(1) Prove Cayley's theorem: Every group is isomorphic to a group of permutations. [5 marks]

(2) Let $G = \mathbb{Z}_4 \oplus \mathbb{Z}_4$. Show that $H = \{(0,0), (2,0), (0,2), (2,2)\}$ and $K = \langle(1,2)\rangle$ are subgroups of G. Determine if G/H and G/K are isomorphic. Justify your answer. [5 marks]

(3) Suppose that there is a homomorphism from a finite group G onto $\mathbb{Z}_{10}$. Prove that G has normal subgroups of index 2 and 5. [5 marks]

(4) Determine a 2-Sylow subgroup of S_4. [5 marks]