Department of Mathematics

Major Examination

MTL 106: Probability and Stochastic Processes

Venue: LH 325 | **Date: 01-05-2017** | **Time 10:30 – 12:30 PM** | **Full Marks 45**

Q1. a) Let $X_1, X_2, \ldots, X_{101}$ be independent $N(0,1)$ variates.

Find the expectation of $\dfrac{\sum_{i=1}^{51} X_i^2}{\sum_{i=51}^{101} X_i^2}$.

Justify each step of your answer, clearly stating any results that you might have assumed

b) If X and Z are independent $N(0,1)$ variates, find the covariance between $X^2 + Z^2$ and $X^2 - Z^2$

[5 + 3 = 8]

Q2. a) Consider a r.v. X with the following distribution: X : $\begin{matrix} -1 & 0 & 1 \\ 1/8 & 3/4 & 1/8 \end{matrix}$

Evaluate $P\{|X - \mu| \geq 2\sigma\}$. Verify whether it obeys Chebyshev's inequality.

b) State and prove the Weak Law of Large numbers.

[3 + 4 = 7]

Q3. a) Let $\{X_n\}$ be a sequence of random variables, and X be another random variable defined on the same Ω. Suppose X and each X_n takes a constant value 5 with probability 1. Show that both $X_n \xrightarrow{P} X$ and $X_n \xrightarrow{L} X$ hold. Can we say the same if X and each X_n takes two values 5 and 10 with equal probabilities? Justify your answer.

b) If $X_1, X_2, X_3, \ldots X_n$ are *i.i.d* Poisson random variables with Mean $= \mu$ and Variance $= \sigma^2$. Find the asymptotic distribution of $S_n = \sum_{i=1}^{n} X_i$.

[2 + 2 + 4 = 8]

Q4. a) Consider a 2-state DTMC with sates 0 and 1. Let $P = \begin{bmatrix} 2/3 & 1/3 \\ 1/2 & 1/2 \end{bmatrix}$ be the one-step transition matrix.

Using spectral decomposition of P compute $p_{kj}^{(100)}$ for $k, j \in \{0, 1\}$.

b) Compare the above with the results obtained for a 2-state DTMC.

[5 + 3 = 8]

Q5. a) Let $\{N(t) \mid t \geq 0\}$ be a Poisson process with parameter λ. Will the process $\{X(t) \mid t \geq 0\}$ defined as $X(t) = N(t + K) - N(K)$, where K is a positive constant, be Wide-Sense Stationary? Justify your answer.

b) State and prove Chapman-Kolmogorov Backward equation with respect to a CTMC explaining each term.

[4 + 3 = 7]

Q6. a) What are the characteristic features of a queuing system? What is an M/M/4/50/LCFS queuing system?

b) Verify Little's Theorem for time point T, where T is exactly 1 minute after the 10^{th} customer arrives for the following queuing system with one server. The process starts at time = 0 when the first customer arrives. After that at the end of every minute a new customer arrives to the system. The serving time for a customer is 2 minutes for customer number 1, 3, 5, ... and 3 minutes for customer number 2, 4, 6, ... Each customer leaves right after being served. Assume there is no delay time as the server attends the next customer.

[2+ 5 = 7]