Probability and Stochastic Processes (MTL106)

MAJOR EXAMINATION – Sem I – 2017-18

Time: 2 hours LH121, 6 – 8 PM November 21, 2017 Full Marks: 43

Q1. Suppose there are two candidates A and B. Suppose further that the fraction of the population who prefer A to B is = 0.5. To run a poll, a pollster selects n=1600 people at random and asks 'Do you support candidate A or candidate B'. Based on the given information answer the following questions:

a) Find out the probability that more than 1200 people vote for person A?

b) Estimate the probability that more than 1200 people vote for person A using Chebyshev inequality.

c) State Central limit theorem (CLT) and use CLT to estimate the probability that more than 1200 people vote for person A.

Final answer can be in terms of $\phi(z)$ *where* $\phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}dt$

d) Is it true that CLT provides better estimate for probability as compared to Chebyshev inequality? You can use the value of $\phi(4)$ as 0.99997 for comparison of probability as estimated by CLT and Chebyshev inequality..

[1+3+3+1 = 8]

Q2. a) Suppose that in the general population of MTL106 students Mathematical and Programming abilities are independently and uniformly distributed in [0, 1]. Students get grade point 8 or above if and only if the sum of their Mathematical and Programming abilities is greater than 1. Among the students who get grade point 8 or above what fraction have Mathematical ability above 0.9?

b) Prove that if a discrete distribution is "memory less" then it has to be Geometric.

[4+ 4 = 8]

Q3. a) You enter a metro station in a big hurry and decide to take the first train that arrives. There are two lines running through this station: one runs every five minutes (line A), the other every three (line B). To be precise, suppose the next arrival of train A is uniformly distributed on the interval [0, 5] minutes and that of train B is on [0, 3] minutes. The two arrivals are independent. How many minutes will you wait on average until you get on a train?

b) If 100 numbers are drawn from the interval (0, 2) find the expected value of the inter-quartile deviation Q3 – Q1, assuming that they follow uniform distribution.

[4+4 = 8]

$3 \times 8\ 4$

$^8C_6\ ^8C_1\ ^8C_2\ ^8C_3\ ^8C_4\ ^8C_5\ ^4C_1\ ^4C_2\ ^4C_3\ ^4C_4$

$P(X \geq s) = \ ^8C_6\ ^8C_7\ ^8C_8$

$P(t \leq X \leq t+s) =$

$8+7$

$P(X \leq s) = P(X \leq t+s \mid X \geq t)$

$P(X \leq s) = \frac{P(t \leq X \leq t+s)}{P(X \geq t)}$

$\frac{1}{256}$ $\frac{256}{4}$

Q4. You throw a six sided ordinary dice until you obtain all the six numbers – 1,2,3,4,5,6. Note that the numbers do not have to come in this specific order, i.e. they can come in any order.

a) Depict and Describe the states of discrete time 7-state Markov chain as well as transition probabilities.
b) Using linearity of expectation, write your random variable as sum of 6 other random variables.
c) Find each individual random variable's expectation and hence the required number of dice throws.

Hint: Think how many dice throws you would need to get $(k+1)^{th}$ number, if you have already seen k (for example 2) of the numbers.

[4 + 1 + 4 = 9]

Q5. a) In a town there are N people out of which one , say X, is the only one suffering from 'Greyscale' – an infectious disease. Contact between any two persons of the population follows a Poisson Process with parameter λ and it is equally likely to involve any pair from the population (i.e the parameter is λ for the ${}^{N}C_2$ pairs). Whenever an uninfected person comes in contact with an infected person, the former becomes infected as well.

i Model this situation as a CTMC by specifying the states and the transition rates amongst them.
ii Find the expected time when all the people become infected.

b) Consider a gas filling station with 4 machines to serve the customers. Suppose it has infinite capacity to hold as many cars as they arrive. If the rate at which a car is gas filled is 15 per hour, and cars arrive at a rate 12 per hour draw the transition diagram of the Queuing process. Justify your answer.

[3 + 3 + 4 = 10]

$e^{-\lambda}$ λ

1.2 + 2.1