Department of Mathematics

MTL 106 (Introduction to Probability Theory and Stochastic Processes)

Minor 1 (I Semester 2015 - 2016)

Time allowed: 1 hour **Max. Marks: 25**

1. (a) Write axiomatic definition of probability.

   (b) Show that the conditional probability $P(A/B)$ satisfies the three axioms of probability.

   (3 + 3 marks)

2. Let X be a random variable such that $P(X=2)=\frac{1}{4}$ and its distribution function is given by

$$F_X(x)=\begin{cases}0, & x<-3\\ \alpha(x+3), & -3\leq x<2\\ \frac{3}{4}, & 2\leq x<4\\ \beta x^2, & 4\leq x<8/\sqrt{3}\\ 1, & x\geq 8/\sqrt{3}\end{cases}.$$

   (a) Find α, β if 2 is the only jump discontinuity of F.

   (b) Compute $P(X<3/X\geq 2)$. (1 + 1 + 2 marks)

3. Suppose the length of a telephone conversation between two persons is a random variable X with cumulative distribution function

$$P(X\leq t)=\begin{cases}0, & -\infty<t<0\\ 1-e^{-0.04t}, & 0\leq t<\infty\end{cases},$$

   where the time is measured in minutes.

   (a) Given that the conversation has been going on for 20 minutes, compute the probability that it continues for at least another 10 minutes.

   (b) Show that, for any $t>0$, $E(X/X>t)=t+25$.

   (3 + 2 marks)

4. Consider a random variable X with $E(X)=1$ and $E(X^2)=1$.

   (a) Find $E[(X-E(X))^4]$ if it exists.

   (b) Find $P(-1/2<X\leq 3)$ and $P(X=0)$.

   (3 + 1 + 1 marks)

5. Suppose that X is a continuous random variable with pdf $f_X(x)=e^{-x}$ for $x>0$. Define $Y=\begin{cases}X, & X<1\\ \frac{1}{X}, & X\geq 1\end{cases}$.

   (a) Discuss whether the distribution of Y is discrete or continuous or mixed type.

   (b) Determine the pmf/pdf as applicable to this case.

   (1 + 4 marks)