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MTL 106 (Introduction to Probability Theory and Stochastic Processes)

Time allowed: 1 hour **Minor 1 Examination** **Max. Marks: 25**

Name: Jitender singh Entry Number: 2016EE10450 Signature: jitendersingh

Multiple Selection Questions: **Section 1** **($2 \times 2 = 4$ marks)**

Each of the following questions 1 and 2 has four options out of which one or more options can be correct. Write A, B, C or D which corresponds to the correct option for the first correct answer followed by space and the next correct answer and so on. **2 marks** is awarded if all correct answers are written, **0 mark** for no answer or partial correct answers or any incorrect answer.

1. Based on the probability concepts, which of the following statements are NOT TRUE?
   (A) Ω is the collection of few possible outcomes of a random experiment.
   (B) $(\Omega, \mathcal{F}, P)$ is a probability space. (C) $\mathcal{F}$ is a σ-field on the subset of Ω.
   (D) P is a measure. Answer: A (2) (2 marks)

2. Which of the following distributions NOT satisfy the memoryless property? (0)
   (A) Exponential (B) Poisson (C) Bernoulli (D) Geometric Answer: ABC (2 marks)

Numeric Type Questions: **Section 2** **($6 \times 2 = 12$ marks)**

Write the answer upto 4 decimal places or in fraction or the expression for the following questions 3 to 8. **2 marks** are awarded if answer is correct, and **0 mark** for no answer or an incorrect answer.

3. Let $\Omega = \{a, b, c, d\}$, $\mathcal{F} = \{\emptyset, \{a\}, \{b, c\}, \{d\}, \{a, b, c\}, \{b, c, d\}, \{a, d\}, \Omega\}$ and P a function from $\mathcal{F}$ to $[0, 1]$ with $P(\{a\}) = \frac{2}{5}$, $P(\{b, c\}) = \frac{2}{7}$ and $P(\{d\}) = \beta$. The value of β such that P to be a probability on $(\Omega, \mathcal{F})$. Answer: $\frac{11}{35}$ (2) (2 marks)

4. Consider a gambler who on each independent bet either wins 1 with probability $\frac{1}{4}$ or losses 1 with probability $\frac{3}{4}$. The gambler will quit either when he or she is winning a total of 10 or after 50 plays. The probability the gambler plays exactly 13 times?
   Answer: 0 (0) (2 marks)

5. The first generation of particles is the collection of offsprings of a given particle. The next generation is formed by the offsprings of these members. If the probability that a particle has k offsprings is p_k, where $p_0 = \frac{1}{2}, p_1 = \frac{1}{3}$ and $p_2 = \frac{1}{6}$. Assume that particles act independently and identically irrespective of the generation. Find the probability that there is no particle in third generation. Answer: 1 (0) (2 marks)

6. Let $\Omega = \{1, 2, 3\}$. Let $\mathcal{F}$ be a σ-algebra on Ω, so that $X(w) = w - 1$ is a random variable. Then, $\mathcal{F}$ is given by
   Answer: $\mathcal{F} = \{\phi, \{1\}, \{2,3\}, \{1,2\}, \{3\}, \Omega\}$ (0) (2 marks)

7. Consider a random variable X with $E(X) = 1$ and $E(X^2) = 1$. What is the value of $P(0.1 < X < 1.3)$? Answer: 1 (2) (2 marks)

8. The moment generating function (MGF) of a random variable X is given by $M_X(t) = \frac{1}{3} + \frac{1}{6}e^t + \frac{1}{4}e^{2t} + \frac{1}{4}e^{3t}$. If μ is the mean and σ^2 is the variance of X, what is the value of $P(\mu < X < \mu + \sigma)$? Answer: $\frac{1}{4}$ (2 marks)

(2)

Comprehensive Type Questions: **Section 3** **(3 × 3 = 9 marks)**

Each of the following questions 9 to 11 has some subparts. Each subparts has four options out of which one is the correct answer. −0.5 **mark** for incorrect answers of 1 mark and −1 **mark** for incorrect answers of 2 marks question. For no answer, **0 mark.**

9. A club basketball team will play a 50-game season. Twenty four of these games are aginst class A teams and 26 are against class B teams. The outcomes of all the games are independent. The team will win each game against a class A opponent with probability 0.4 and it will win each game against a class B opponent with probability 0.6. Let X_A and X_B denote, respectively, the number of victories against class A and class B teams. Let X denote its total victories in the season.

   (a) What is the distributions of X_A? 1
   (A) Poisson (B) Bernoulli (C) Geometric (D) Binormail Answer: D (1 mark)

   (b) What is the relationship between X_A, X_B and X?
   (A) $X = -X_A - X_B$ (B) $X = X_B - X_A$ (C) $X = X_A - X_B$ (D) $X = X_A + X_B$
   Answer: D (1 mark) 1

   (c) What is the distribution of X?
   (A) Geometric (B) Poisson (C) Binormail (D) Bernoulli Answer: (1 mark)

10. Let X be a continuous type random variable with strictly increasing CDF F_X.

   (a) Which is the one of following distributions of X?
   (A) Standard normal (B) Uniform $(0,1)$ (C) Gamma (D) Exponential 1
   Answer: A (1 mark)

   (b) What is the type of distribution have the random variable $Y = -\ln(F_X(X))$?.
   (A) Standard normal (B) Gamma (C) Exponential (D) Uniform $(0,1)$ 2
   Answer: C (2 marks)

11. Let X be a continuous type random variable having the pdf $f(x) = \begin{cases} 0, & -\infty < x \le 0 \\ \frac{1}{2}, & 0 < x \le 1 \\ \frac{1}{kx^2}, & 1 < x < \infty \end{cases}$ 1

   (a) Find k? (A) 4 (B) 3 (C) 5 (D) 2 Answer: D (1 mark)

   (b) Find the pdf of $Y = \frac{1}{X}$?

   (A) $f(y) = \begin{cases} 0, & -\infty < y \le 0 \\ \frac{1}{2}, & 0 < y \le 1 \\ \frac{1}{3y^2}, & 1 < y < \infty \end{cases}$ (B) $f(y) = \begin{cases} 0, & -\infty < y \le 0 \\ \frac{1}{2}, & 0 < y \le 1 \\ \frac{1}{4y^2}, & 1 < y < \infty \end{cases}$

   (C) $f(y) = \begin{cases} 0, & -\infty < y \le 0 \\ \frac{1}{2}, & 0 < y \le 1 \\ \frac{1}{2y^2}, & 1 < y < \infty \end{cases}$ (D) $f(y) = \begin{cases} 0, & -\infty < y \le 0 \\ \frac{1}{4}, & 0 < y \le 1 \\ \frac{3}{4}, & 1 < y < 2 \end{cases}$ 2

   Answer: C (2 marks)