43

MTL 106 (Introduction to Probability Theory and Stochastic Processes)

Time allowed: 2 hours Major Examination Max. Marks: 51

Name: Sanket Gandhi Entry Number: 2017TT10896 Signature: Gandhi

Multiple Selection Questions: **Section 1** (10 × 1 = 10 marks)

Each of the following questions 1 to 10 has four options out of which one or more options can be correct. Write A, B, C or D which corresponds to the correct option for the first correct answer followed by space and the next correct answer and so on. **1 mark** is awarded if all correct answers are written, **0 mark** for no answer or partial correct answers or any incorrect answer.

1. Let $\Omega = \{a, b, c\}$. If $F_1 = \{\emptyset, \{a\}, \{b, c\}, \Omega\}$, $F_2 = \{\emptyset, \{a, c\}, \{b\}, \Omega\}$ and $F_3 = \{\emptyset, \{a, b\}, \{c\}, \Omega\}$ are three σ-fields on Ω. Then, which of the following are σ-fields
(A) $F_1 \cap F_2 \cap F_3$ (B) $F_1 \cup F_2 \cup F_3$ (C) $F_1 \cap F_2$ (D) $F_1 \cup F_2$ Answer: A,B,C 1 4

2. Which of the following distributions do NOT satisfy the memoryless property?
(A) Normal (B) Geometric (C) Exponential (D) Poisson Answer: ~~B~~ D,A

3. If X and Y are independent random variables, then
(A) Var $(X+Y)$ = Var (X) - Var (Y) (B) Var $(X-Y)$ = Var (X) + Var (Y)
(C) Var $(X+Y)$ = Var (X) + Var (Y) (D) Var $(X-Y)$ = Var (X) - Var (Y)
Answer: B, C

4. Let X and Y be two random variables. Suppose a and b are constants. Then,
(A) Cov $(aX+b, Y) = a^2$ Cov $(X, Y) + b$ (B) Cov $(aX+b, Y)$ = a Cov (X, Y)
(C) Cov $(aX+b, Y)$ = Cov $(Y, aX+b)$ (D) Cov $(aX+b, Y)$ = a Cov $(X, Y)+b$ Answer: B, C

5. Let X_1, X_2, X_3 be three rvs such that Cor(X_1, X_2)=Cor(X_2, X_3)=Cor(X_1, X_3)=ρ where Cor denotes correlation coefficient. Then, which of the following statements are FALSE?
(A) $\rho = -1$ (B) X_1, X_2, X_3 are necessarily independent (C) $\rho \geq 0$ (D) $\rho < 0$ Answer:

Space for Rough Work

$$\frac{\text{cov}(X_1, X_2)}{\sqrt{\text{Var}X_1}\ \cancel{\sqrt{X_2}}} = \frac{\text{cov}(X_2, X_3)}{\cancel{\sqrt{\text{Var}X_2}}\ \sqrt{\cdot}}$$

$$\frac{P(X > 5)}{(X > 4)} = \frac{p^5 (1-p)^5 p}{(1-p)^3 p}$$

$$\frac{\int_k^\infty e^{-z^2}}{\int_{k-1}^\infty e^{-z^2}}$$

$$(1-p)^5 \cancel{p}\ \frac{1}{\cancel{1-p}} \quad (1-p)^5 p + (1-p)^6 p + (1-p)^7 p .$$

$$(1-p)^5 \cancel{p} \left[\frac{1}{\cancel{p}} \right] \quad (1-p)$$

$$\boxed{(1-p)^5} \qquad (1-p)\ 4$$

6. Which are the following transition probability matrices for the irredicible Markov chains

(A) $\begin{pmatrix} 0 & 1 & 0 \\ 0.3 & 0.3 & 0.4 \\ 1 & 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 1 & 0 \\ 0.3 & 0.3 & 0.4 \\ 0 & 1 & 0 \end{pmatrix}$ (C) $\begin{pmatrix} 0 & 0.4 & 0.6 \\ 0.3 & 0.3 & 0.4 \\ 0 & 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & 0.4 & 0.6 \\ 0.3 & 0.3 & 0.4 \\ 1 & 0 & 0 \end{pmatrix}$

Answer: AB, D

7. Which are the following generator matrices for the ergodic Markov chains

(A) $\begin{pmatrix} -1 & 0.4 & 0.6 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix}$ (B) $\begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$ (C) $\begin{pmatrix} -1 & 0.4 & 0.6 \\ 0.3 & -0.7 & 0.4 \\ 1 & 0 & -1 \end{pmatrix}$ (D) $\begin{pmatrix} -1 & 0 & 1 \\ 0.3 & -0.7 & 0.4 \\ 1 & 0 & -1 \end{pmatrix}$

Answer: B, C, D

3

8. Assume the life times of $N = 100$ soldiers are iid rvs following an exponential distribution with parameter μ, then the process of the number of surviving soldiers by time t, $\{X(t), t \geq 0\}$, is a pure death process with death rates $\mu_i = i\mu$, $i = 1, 2, \ldots, N$. Assume that, $X(0) = N$. Then, (A) state i, $0 \leq i \leq N-1$ is a transient (B) state i, $1 \leq i \leq N$ is a transient (C) state 0 is an absorbing state (D) state i, $0 \leq i \leq N$ is a transient

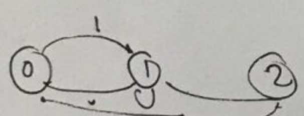
Answer: B, C

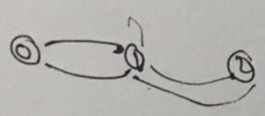
9. Let $\{X_n, n = 0, 1, 2, \ldots\}$ be a DTMC with infinite state space S and $i \in S$ is an absorbing state. Which of the following statements are FALSE?
(A) State i is a +ve recurrent. (B) State i is a null recurrent.
(C) The period of state i, $d_i = 1$. (D) The mean recurrence time of state i, $\mu_i = \infty$.

Answer: A, C

10. Consider a $M/M/5/\infty$ queueing model where $X(t)$ denotes the number of customers in the system at any time t. Then, the number of customers undergoing service at any time t is
(A) 5 (B) $\min\{X(t), 4\}$ (C) $\min\{X(t), 5\}$ (D) $\max\{X(t), 5\}$ Answer: C

Space for Rough Work





$1000 \times 0.25 + 2000 \times 0.5$

$250 + 1000$

$E(X(t)) =$

$E\left[(A_0 + A_1 t + A_2 t^2)(A_0 + A_1 s + A_2 s^2)\right]$

$E\left[A_0^2 + A_0 A_1 s + \cancel{A_0 A_1 t} + \cancel{A_0 A_2 s^2} + \cancel{A_0 A_2 t^2} + A_1^2 st + A_1 A_2 s^2 t + \cancel{A_1 A_2} t^2 s + A_2^2 t^2 s^2\right.$

$1 + st + t^2 s^2$

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Short Answer Type Questions: **Section 2** (10 × 2 = 20 marks)

Write the answer upto 4 decimal places (D) or in fraction (F) or the expression (E) for the following questions 11 to 20. **2 marks** are awarded if answer is correct, and **0 mark** for no answer or partial correct answer or an incorrect answer.

10

11. Let $C_i, i = 1, 2, \ldots, k$ be the partition of sample space Ω. For any events A and B, find $\sum_{i=1}^{k} P(C_i/B)P(A/(B \cap C_i))$.
Answer(E): $\cancel{P(A)}\ \frac{P(A \cap B)}{P(B)} = P(A/B)$

12. Let X be a discrete type rv with moment generating function $M_X(t) = a + be^{2t} + ce^{4t}$, $E(X) = 3$, $\text{Var}(X) = 2$. Find $E(2^X)$?
Answer(E): $\cancel{\frac{49}{4}}$ $\frac{89}{8}$

13. Two points are selected at random in the interval $[0, 1]$. The probability that the sum of their squares is less than 1 is given by
Answer(F/D): $1 - \frac{\pi}{4}$

14. Suppose that, we choose a point (X, Y) uniformly at random in E where $E = \{(x, y) \mid |x| + |y| \leq 1\}$. Then, the joint pdf of (X, Y) is given by
Answer(E): $f_{XY}(x,y) = \begin{cases} \frac{1}{2} & 0 \le x, y \le 1 \\ 0 & \text{otherwise} \end{cases}$

15. Consider Bacteria reproduction by cell division. In any time t, a bacterium will either die (with probability 0.25), stay the same (with probability 0.25), or split into 2 parts (with probability 0.5). Assume bacteria act independently and identically irrespective of the time. Given that there are 1000 bacteria in the population at time $t = 50$, what is the expected number of bacteria at time $t = 51$.
Answer(F/D): 1250

16. Let $X(t) = A_0 + A_1 t + A_2 t^2$, where $A_i's$ are uncorrelated random variables with mean 0 and variance 1. Find the covariance function of $X(t)$.
Answer(E): $1 + st + s^2t^2$

Space for Rough Work

$\frac{3}{4} \times 16 + 1 \times \frac{1}{4}$

$\frac{49}{4}$

$b + c =$ $8a = 2$ $3 = 9a + 6 + c$ $a = 1/4$ $2b + 4c = 3$

$ae^{0t} + be^{2t} + c^{4t}$

$P\left(\frac{C_i \cap B}{B}\right)$ $\frac{P(C_i \cap B)}{P(B)}$ $\frac{P(A) \cap B \cap C_i)}{\cancel{P(B \cap C_i)}}$

$2b + 4c = 3$ $b + c = 3/4$

$\frac{P(A \cap B)}{P(B)}$ $2b + 2c =$ $b + c = \frac{3-2c}{2}$

$E(x^2) = 11$ $3, 1, 1$

$\frac{3-2c}{2} = 3/4\ 2$

$11 = 9a + b + 4c$

$6 - 4c = 3$

$4c = 3$

9 $9 - 36b + 4b + 3 - 8b = 44$

$-40b$

$a + b + c = 1$ $9a + b + c = 11$

$a = \frac{4 - 2b - 3}{4}$

$C = \frac{3 - 2b}{4}$ $b + c = 1 - a$ $c = 3/4$

$a = \frac{1 - 2b}{4}$ $a + \frac{\cancel{4b + 3 - 2b}}{4} = 1$ $2b + 3$ $8a + 1 = 11$ $b = 0$

$\frac{9}{4}(1 - 4b) + b + (3 - 2b) = 11$

$\boxed{a = \frac{11}{8}}$

17. Consider a DTMC with transition probability matrix $\begin{pmatrix} 0.4 & 0.6 & 0 \\ 0.4 & 0 & 0.6 \\ 0 & 0.4 & 0.6 \end{pmatrix}$. The stationary distribution for this Markov chain is given by

Answer(E): $\cancel{\left(\frac{4}{19} \quad \frac{9}{19} \quad 6/19\right)}$ $\begin{array}{ccc} \pi_0 & \pi_1 & \pi_2 \\ \frac{4}{19} & \frac{6}{19} & \frac{9}{19} \end{array}$

18. Consider a CTMC with $Q = \begin{pmatrix} -5 & 3 & 2 \\ 1 & -3 & 2 \\ 2 & 4 & -6 \end{pmatrix}$ and initial distribution $(0, 0, 1)$. Find $P(\tau > t)$ where τ denotes the first transition time of the Markov chain. Answer(E): e^{-6t}

19. Suppose the people immigrate into a territory at time homogeneous Poisson process with parameter $\lambda = 1$ per day. Let $X(t)$ be the number of people immigrate on or before time t. What is the probability that the elapsed time between the 200th and 201th arrival exceeds two days? Answer(E): e^{-2}

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20. Consider an $M/M/1/2$ queueing system with $\lambda = 3$ per minute and $\mu = 6$ per minute. Then, the steady state probabilities the queueing model are give by

Answer(E): $\left(\underset{\pi_0}{4/7} \quad \underset{\pi_1}{2/7} \quad \underset{\pi_2}{1/4} \quad 0 \ldots\right)$ i.e. $\pi_K = \begin{cases} (4/7)(1/2)^K & ; K = 0,1,2 \\ 0 & ; \text{otherwise} \end{cases}$

Space for Rough Work

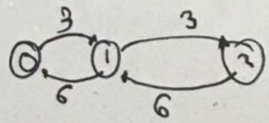
$6e^{-6t}$

$0.4\pi_0 + 0.4\pi_1 = \pi_0$

e^{-2}

$\pi = \pi Q$ $\quad 0.6\pi_0 + 0.4\pi_2 = \pi_1$

$0.6\pi_1 + 0.6\pi_2 = \pi_2$



$0.6[1 - \pi_0] = \pi_2$ $\qquad \pi_2 + \pi_0 + \pi_1 = 1$

$\therefore 0.6 = \pi_2 + 0.6\pi_0$ $\qquad \pi_0 = \dfrac{1}{1 + \frac{1}{2} + \frac{1}{4}}$

$\pi_0 + 0.4\pi_2 = 0.4$ $\qquad 4/7 \times 1/2$

$\dfrac{1}{3/2 + 1/4}$

$0.24 - 0.24\pi_0 = 0.4 - \pi_0$

0.6

$\dfrac{6}{4} + 1/4$

$0.76\pi_0 = 0.16$ $\qquad 0.6\left[\dfrac{15}{19}\right]$

$\pi_0 = \dfrac{4}{19}$ $\qquad \dfrac{9.0}{19}$ $\qquad 4/7$

Subjective Type Questions: **Section 3** (3 × 5 and 6 = 21 marks)

Write the answer in the same page provided for the questions 21 and 24. **Full marks** are awarded if all the steps are correct, and **partial marks** for an incorrect answer with wrong steps.

21. Let $(\Omega, \Im, P) = ([0,1], \mathcal{B}(\mathbb{R}) \cap [0,1], U([0,1]))$. Let $\{X_n, n = 1, 2, \ldots\}$ be a sequence of random variables with $X_n \overset{d}{=} U\left(\left[\frac{1}{2} - \frac{1}{n}, \frac{1}{2} + \frac{1}{n}\right]\right)$. Prove or disprove that $X_n \xrightarrow{d} X$ with $P(X = \frac{1}{2}) = 1$. (5 marks)

Answer:

now $X_n \xrightarrow{d} X$ i.e. ~~$F_{X_n} \to F_X$~~ $F_{X_n} \to F_X$

now $F_X(x) = \begin{cases} 1 & \text{if } x \geq 1/2 \\ 0 & \text{if } x < 1/2 \end{cases}$

now $F_{X_n}(x) = \begin{cases} 1 & \text{if } x \geq 1/2 + 1/n \\ \dfrac{x - (1/2 - 1/n)}{2/n} & \text{if } \frac{1}{2} - \frac{1}{n} \leq x < 1/2 + 1/n \\ 0 & \text{otherwise} \end{cases}$

3

now $\lim_{n \to \infty} F_{X_n}(x) = \begin{cases} \lim_{n\to\infty} 1 & x \geq 1/2 + 1/n \\ \lim_{n\to\infty} \dfrac{n(x - 1/2) + 1}{2} & \text{if } 1/2 - 1/n \leq x < 1/2 + 1/n \\ \lim_{n\to\infty} 0 & \text{otherwise} \end{cases}$

now $\lim_{n\to\infty} 1 = 1$ and $\lim_{n\to\infty} 0 = 0$

$\lim_{n\to\infty} \dfrac{n(x - 1/2) + 1}{2} = \dfrac{n(0) + 1}{2} = 1/2$

now as $n \to \infty$

$1/2 - 1/n \leq x < 1/2 + 1/2$

$\Downarrow$

$1/2 \leq x < 1/2 + \boxed{x = 1/2}$

now no matter how large n we take $F_{X_n}(1/2) = 1/2$

$\therefore F_{X_n} \overset{d}{\not\to} F_X$

hence $X_n \overset{d}{\not\to} X$

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22. In a communication system, the carrier signal at the receiver is modeled by $Y(t) = X(t)\cos(2\pi wt + \Theta)$ where $\{X(t), t \geq 0\}$ is a zero-mean and wide- sense stationary process, Θ is a uniform distributed random variable with interval $(-\pi, \pi)$ and w is a positive constant. Assume that, Θ is independent of the process $\{X(t), t \geq 0\}$. Is $\{Y(t), t \geq 0\}$ wide-sense stationary? Justify your answer.

(5 marks)

Answer:

① $E(Y(t)) = E(X(t)\cos(2\pi\omega t+\theta))$

$= E(X(t)) \cdot E(\cos(2\pi\omega t+\theta))$

$= 0$

i.e $E(Y(t))$ is independent of t.

② $E(Y^2(t)) = E(X^2(t)\cdot \cos^2(2\pi\omega t+\theta))$

$= E(X^2(t)) \cdot E(\cos^2(2\pi\omega t+\theta))$

$= \frac{1}{2}E(X^2(t)) \Rightarrow$ independent of t [$E(X^2(t))$ is independent of t]

finite

finite.

now $E(\cos^2(2\pi\omega t+\theta)) = \int_{-\pi}^{\pi} \frac{1}{2\pi}\cos^2(2\pi\omega t+\theta)\,d\theta = \int_{-\pi}^{\pi} \frac{[\cos(4\pi\omega t+2\theta)+1]}{4\pi}\,d\theta$

$= 1/2$

4

now

③ ~~cov(X(t),~~ $\mathrm{cov}(Y(t), Y(s)) = E[(Y(t)-0)(Y(s)-0)]$

$= E[X(t)\cos(2\pi\omega t+\theta)\cdot X(s)\cos(2\pi\omega s+\theta)]$

$= E[X(t)X(s)]\cdot E\left[\frac{1}{2}\left[\cos\left(\frac{2\omega\pi(s+t)+2\theta}{2}\right)+\cos\left(\frac{2\omega\pi(t-s)}{2}\right)\right]\right]$

now $\mathrm{cov}(X(t), X(s)) = E[(X(t)-0)(X(s)-0)] = f|t-s| \longrightarrow X(t) \to$ wide sense stationary

$\mathrm{cov}(Y(t),Y(s)) = f|t-s|\,\frac{1}{2}\left[E\left(\cos\left(\frac{2\pi\omega(t+s)+2\theta}{2}\right)\right)+\cos(\omega\pi(t-s))\right]$

$A = \int_{-\pi}^{\pi} \cos(\pi\omega(t+s)+\theta)\,d\theta$

$= \sin(\omega\pi(t+s)+\pi) - \sin(\omega\pi(t+s)-\pi)$

$= 0$

$\mathrm{cov}(Y(t),Y(s)) = f|t-s|\,\frac{1}{2}\cdot\cos(\omega\pi(t-s))$

$\boxed{\text{hence } Y(t) \text{ is wide Sense Stationary}}$

23. Consider a time-homogeneous CTMC $\{X(t), t \geq 0\}$ which takes the value 0 and 1 with probability $\pi_0(t)$ and $\pi_1(t)$ at any time t, respectively. Also $Prob\{X(t+\Delta t) = 1/X(t) = 0\} = \alpha\Delta t + o(\Delta t)$ and $Prob\{X(t+\Delta t) = 0/X(t) = 1\} = \beta\Delta t + o(\Delta t)$. As $\Delta t \to 0$, $o(\Delta t) \to 0$. Assume that, α and β are positive constants. Assume $\pi_0(0) = 1$.

(a) Draw the state transition diagram for the Markov chain $\{X(t), t \geq 0\}$.

(b) Write the Kolmogorov forward equations for the Markov chain $\{X(t), t \geq 0\}$.

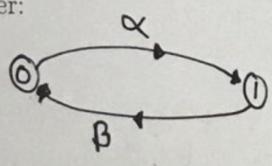
(c) Derive the transient or time-dependent probability distribution of the Markov chain.

(2 + 1 + 3 marks)

Answer:

6

a) now



b) now $Q = \begin{bmatrix} -\alpha & \alpha \\ \beta & -\beta \end{bmatrix}$ and $P(t) = \begin{bmatrix} P_{00}(t) & P_{01}(t) \\ P_{10}(t) & P_{11}(t) \end{bmatrix}$

∴

and $P'(t) = \begin{bmatrix} P'_{00}(t) & P'_{01}(t) \\ P'_{10}(t) & P'_{11}(t) \end{bmatrix}$

∴ kolmogrorov forward equation is $\boxed{P'(t) = P(t)\,Q}$

~~full P(t) matrix required.~~

c) now $\pi'(t) = \pi(t)\,Q$

$\therefore \begin{bmatrix} \pi_0'(t) & \pi_1'(t) \end{bmatrix} = \begin{bmatrix} \pi_0(t) & \pi_1(t) \end{bmatrix} \begin{bmatrix} -\alpha & \alpha \\ \beta & -\beta \end{bmatrix}$

$\therefore \pi_0'(t) = -\alpha\pi_0(t) + \beta\pi_1(t)$

$\pi_1'(t) = \alpha\pi_0(t) - \beta\pi_1(t)$

now $\pi_0(t) + \pi_1(t) = 1$

$\therefore \pi_0'(t) = -(\alpha+\beta)\pi_0(t) + \beta$

now $\pi_1(t) = 1 - \pi_0(t)$

$\therefore \pi_0'(t) + (\alpha+\beta)\pi_0(t) = \beta$

$\boxed{\pi_1(t) = \frac{\alpha}{\alpha+\beta}\left[1 - e^{-(\alpha+\beta)t}\right]}$

$\therefore \pi_0(t)\cdot e^{(\alpha+\beta)t} = \int \beta e^{(\alpha+\beta)t}\,dt$

$\boxed{\pi_0(t) = \frac{\left[\beta + \alpha e^{-(\alpha+\beta)t}\right]}{\alpha+\beta}}$

24. A dental surgery has two operation rooms. The service times are assumed to be independent, exponentially distributed with mean 15 minutes. Mr. Ram arrives when both operation rooms are empty. Mr. Rajesh arrives 10 minutes later while Ram is still under medical treatment. Another 20 minutes later Mr. Ajit arrives and both Ram and Rajesh are still under treatment. No other patient arrives during this 30 minute interval.

(a) What is the probability that the medical treatment will be completed for Ajit before Ram?

(b) Derive the distribution function of the waiting time in the system for Ajit?

(2 + 3 marks)

Answer:

a) As exponential distribution has memory less property we say when when ~~Ram~~ Ajit arrives again new clock starts. [below time starts after ajit arrival]

let $X_1 \sim$ time for Ajit , $X_2 \sim$ time for Ram , $X_3 \sim$ time for rajesh

now $P(X_1 + X_3 < X_2)$

now $X_i \sim exp(1/15)$ $\therefore$ $X_1 + X_3 \sim Gama(2, 1/15)$

$$P(X_1 + X_3 < X_2) = \int_0^\infty \int_0^t \frac{x^1 \cdot (1/15)^2 e^{-1/15 x}}{1!} \left(\frac{1}{15}\right) e^{-1/15 t} \, dx \, dt$$

$$= \int_0^\infty \frac{1}{15}\left[e^{-1/15 t} - e^{-2/15 t} - \frac{t}{15} e^{-2/15 t}\right] dt$$

$$= \frac{1}{15}\left[15 - \frac{15}{2} - \frac{15}{4}\right] = 1/4$$

2

b) now waiting time for ajit will be $\sim \min\{X_2, X_3\}$

$\therefore$ $P(\text{waiting time} > t) = P(X_2 > t, X_3 > t) = P(X_2 > t) P(X_3 > t)$ } independent both

$= e^{-1/15 t} \cdot e^{-1/15 t}$

$= e^{-2/15 t}$

3

$\therefore$ $\boxed{\text{waiting time} \sim exp(2/15)}$

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$a+b+c=1$

$2a+4b=3$

$\lambda^2 x e^{-\lambda x}$

$9a+b+c=2$

$\left[\frac{\lambda^2 x e^{-\lambda x}}{-\lambda}\right]_0^t$

$9a-a+1=2$

$b+c=\frac{3-2c}{2}$

$-\int_0^t \frac{\cancel{x} e^{-\lambda x}}{+\lambda^2}$

$a=1/8$

$\lambda^2 x e^{-\lambda x}$

$3-2c=7/4$

$b+c=7/8$

$-\lambda t e^{-\lambda t} - e^{-\lambda t}+1$

$\left[\frac{\lambda^2 x e^{-\lambda x}}{-\lambda}\right]_0^t + -\int_0^t \lambda \frac{e^{-\lambda x}}{\cdot} dx$

$3-\frac{7}{4}=2c$

$-\lambda t e^{-2\lambda t} - e^{-2\lambda t} + e^{-\lambda t}$

$-\lambda t e^{-tx} + 1 - e^{-\lambda t}$

$5/4$

$\lambda\left[-\lambda t e^{-2\lambda x} + e^{-\lambda x} - e^{-2\lambda x}\right]$

$c=5/8$

$b=2/8$

$\lambda\left[\frac{1}{\lambda}-\frac{1}{2\lambda}-\left[\frac{\lambda t}{-2\lambda}e^{-2\lambda x}+\int\frac{1}{-4\lambda}e^{-2\lambda x}\right.\right.$

$\frac{1}{\lambda}-\frac{1}{2\lambda}$ $\quad$ $\frac{1}{\lambda}-\frac{1}{2\lambda}-\frac{1}{4\lambda}$

$\frac{2}{8}\cdot 2^2+\frac{1}{8}+\frac{5}{8}\times 16$

$\frac{-\lambda t e^{-2\lambda t}}{-2\lambda} + -\int\frac{e^{-2t}}{2}$

$11+\frac{1}{8}$

$\frac{2}{2}+\frac{1}{8}+\frac{5}{8}10$

$\frac{89}{8}$

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Space for Rough Work

a, b

$a^2 + b^2 < 1$

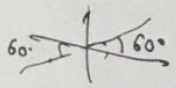
$a^2 < \sqrt{1-b^2}$

$\int_0^1 \frac{1}{1}\left[1-\frac{\sqrt{1-b^2}}{0}\right.$

$\int_0^1 1 - \sqrt{1-b^2}$

$\cancel{\mathcal{Z}} - \frac{\pi}{4}$

$1 - \frac{\pi}{4}$



$\int \sqrt{1-b^2}\; db$

$b = \sin\theta$

$\int \cos\theta . \cos\theta \, d\theta.$

$\int \cos^2\theta \, d\theta$

$\int \frac{\cos 2\theta + 1}{2} \, d\theta$

$\frac{\cos 2\theta + 1}{2} \quad - \frac{\sin 2\theta}{4} + \theta/2$

$\frac{\cos(4\pi\omega t + 2\theta)}{4\pi} + \frac{2\theta}{4\pi}$ $1/2$

$\cos(4\pi\omega t + \pi) - \cos(4\pi\omega t - \pi)$

$\frac{\beta + \alpha}{\alpha + \beta}$

$0.4\pi_0 + 0.4\pi_1 = \pi_0$

$0.6\pi_0 + 0.4\pi_2 = \pi_1$

$0.6\pi_1 + 0.6\pi_2 = \pi_2$

$\therefore 0.6 - 0.6\pi_0 = \pi_2$

$0.6 = \pi_2 + 0.6\pi_2$

$0.4 = \pi_0 + 0.4\pi_2$

$0.6\left[1 - \frac{4}{19}\right.$

$0.24 - 0.24\pi_0 = 0.4 - \pi_0$

$0.76\pi_0 = 0.16$

$\frac{15}{19} \times 0.6 \quad 3 \quad 9.0 \quad 3$

$\pi_0 = \frac{4}{19}$