# Department of Mathematics
## MAL 250 (Introduction to Probability Theory and Stochastic Processes)
## Minor II (I Semester 2013 - 2014)

**Time allowed: 1 hour** **Max. Marks: 27**

1. Let $X$ and $Y$ be random variables with joint *pdf* given by:

$$f(x,y) = \begin{cases} \frac{24x^2}{y^3} & 0 < x < 1;\ y > 2 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the marginal density function of $X$.

   (b) Calculate $P\left(X < \frac{1}{2}/Y > 6\right)$.

(2 + 3 marks)

2. Let $A, B$ and $C$ be independent random variables each with uniform distributed on interval $(0,1)$. What is the probability that $Ax^2 + Bx + C = 0$ has real roots? (5 marks)

3. Let $X$ be a random variable which is uniformly distributed over the interval $(0,1)$. Let $Y$ be chosen from interval $(0, X]$ according to the pdf

$$f(y/x) = \begin{cases} \frac{1}{x} & 0 < y \leq x \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find $E(Y^k/X)$ for any fixed positive integer $k$.

   (b) Find the characteristic function of $Y$.

(2 + 3 marks)

4. Using MGFs, find the limit of Binomial distribution with parameters $n$ and $p$ as $n \to \infty$ such that $np = \lambda$ so that $p \to 0$. (4 marks)

5. Does the random variable $X$ exist for which

$$P\left[\mu - 2\sigma \leq X \leq \mu + 2\sigma\right] = 0.6$$

   Justify your answer. (2 marks)

6. (a) State central limit theorem.

   (b) Prove the central limit theorem with the assumption of the sequence of iid random variables.

   (c) Let $\{X_i, i = 1, 2, \ldots\}$ be a sequence of iid random variables with mean 10 and standard deviation 4. This sequence of random variables form a population. A sample of size 100 is taken from this population. Find the approximate probability that the sample mean of these 100 observations is less than 9. $(P(Z < -2.5) = 0.0062,\ \ P(Z < -2.0) = 0.0228,\ \ P(Z < -1.5) = 0.0668)$

(2 + 2 + 2 marks)