Department of Mathematics
MTL 106 (Introduction to Probability Theory and Stochastic Processes)
Minor II (II Semester 2015 - 2016)

Time allowed: 1 hour **Max. Marks: 25**

1. Roll a fair die until you get a 6. Let X be the number of 1's you got before you got 6. Let Y be the number of rolls (including the 6). Find the probability mass function of Y. Find the joint probability mass function of (X, Y). Also, determine $P(| X - Y | \geq 2)$. (1 + 3 + 2 marks)

2. Let (X, Y) ba a two-dimensional random variable with joint pdf

$$f(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\frac{1}{2}Q(x,y)}, \quad -\infty < x, y < \infty$$

where $-\infty < \mu_1, \mu_2 < \infty$, $\sigma_1, \sigma_2 > 0$, $| \rho | < 1$ and

$$Q(x,y) = \frac{1}{(1-\rho^2)} \left[\left(\frac{x-\mu_1}{\sigma_1} \right)^2 - 2\rho \left(\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} \right) + \left(\frac{y-\mu_2}{\sigma_2} \right)^2 \right]$$

Find the conditional pdf $f_{Y/X}(y/x)$.

(4 marks)

3. Let X, Y, Z be iid random variables, each having uniform distribution in the interval $(1, 2)$. Find $Var\left(\frac{4X}{3Y} + \frac{3Y}{2Z}\right)$.

(4 marks)

4. Let X and Y be two random variables represent two observations of a signal corrupted by noise. Assume that, these two random variables have the same mean μ and variance σ^2. The *signal-to-noise-ratio (SNR)* of the observation X or Y is defined as the ratio $SNR_X = \frac{\mu^2}{\sigma^2}$. A system designer chooses the average strategy, whereby he/she constructs a new random variable $S = \frac{X+Y}{2}$.
   (a) Find the correlation coefficient $\rho_{X,Y}$ when the SNR of S is 1.5 times that of the individual observations, i.e., $SNR_S = (1.5)SNR_X$.
   (b) Under what condition on $\rho_{X,Y}$ can the average strategy result in an SNR_S that is arbitrarily high?

(3 marks)

5. (a) State central limit theorem.
   (b) Prove the central limit theorem with the assumption of the sequence of iid random variables and without the assumption of existence of MGF of the sequence of random variables.
   (c) Suppose each day of the year, the value of a particular stock, increases by one percent with probability 0.5, remains the same with probability 0.4, and decreases by one percent with probability 0.1. Changes on different days are independent. Consider the values after one year (365 days), beginning with one unit of stock. Using CLT, find the probability the stock at atleast triples in value. (Use $P(Z > 2.77) = 0.003$)

(2 marks)

(2 + 2 + 2 marks)