Department of Mathematics
MTL 106/MAL 250 (Introduction to Probability Theory and Stochastic Processes)
Minor II (II Semester 2014 - 2015)

Time allowed: 1 hour **Max. Marks: 25**

1. Reliability, denoted by $R(t)$, is defined as the probability that the component or system experiences no failures during the time interval 0 to t. An aircraft has four engines, each of which has an exponential distributed failure time with parameter λ.

   (a) For a successful flight at least two engines should be operating. Find the reliability $R(t)$ and expected lifetime of the aircraft. (1 + 2 marks)

   (b) Find the reliability and expected lifetime if the aircraft needs at least one operating engine on either side for a successful flight. (1 + 1 marks)

2. Let X and Y be iid random variables each $N(0,1)$ distributed. Find the joint probability density function of (U,V) where $U = 2X - 5Y$ and $V = X - 4Y$. (5 marks)

3. Suppose that a signal X, standard normal distributed, is transmitted over a noisy channel so that the received measurement is $Y = X+W$, where W follows normal distribution with mean 0 and variance σ^2 is independent of X. Find $f_{X/y}(x/y)$ and $E(X \mid Y = y)$. (3 + 2 marks)

4. Amit and Supriya agreed to meet between 7:00 PM and 8:00 PM in a restaurant, with the understanding that each will wait no longer than 15 minutes for the other. Let X be the random variable representing the arrival time (in minutes) of Amit and let Y be the random variable representing the arrival time (in minutes) of Supriya. Suppose that X and Y are independent and identically distributed each with uniform distribution over the interval [7, 8]. What is the probability that they will meet? (5 marks)

5. Consider polling of n voters and record the fraction S_n of those polled who are in favour of a particular candidate. If p is the fraction of the entire voter population that supports this candidate, then $S_n = \frac{X_1+X_2+\ldots+X_n}{n}$, where X_i are independent Bernoulli distributed random variables with parameter p. How many voters should be sampled so that we wish our estimate S_n to be within 0.01 of p with probability at least 0.95? (5 marks)