Department of Mathematics

MTL 106 (Introduction to Probability Theory and Stochastic Processes)

Minor II (I Semester 2015 - 2016)

Time allowed: 1 hour **Max. Marks: 25**

1. Consider a transmitter sends out either a 0 with probability p, or a 1 with probability $(1-p)$, independently of earlier transmissions. Assume that the number of transmissions within a given time interval is Poisson distributed with parameter λ. Find the distribution of number of 1's transmitted in that same time interval? (4 marks)

2. Romeo and Juliet have a date at a given time, and each, independently, will be late by an amount of time, denoted by X and Y respectively, that is exponentially distributed with parameter λ. Find the pdf of, $X - Y$, the difference between their times of arrival? (5 marks)

3. Consider the metro train arrives at the station near your home every quarter hour starting at 5:00 AM. You walk into the station every morning between 7:10 and 7:30 AM, with the time in this interval being a uniform random variable, that is $\mathcal{U}([7:10, 7:30])$. 5:15 7:15

   (a) Find the distribution of time you have to wait for the first train to arrive? (4 marks) 7:

   (b) Also, find its mean waiting time? (2 marks)

4. (a) Let X and Y be two identically distributed random variables with $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$ exist. Prove or disprove that $\mathrm{Var}\left(\frac{X+Y}{2}\right) \leq \mathrm{Var}\,(X)$. (3 marks)

   (b) Let X and Y be i.i.d. random variables each having a standard normal distribution. Calculate $E[(X+Y)^4/(X-Y)]$. (2 marks)

5. Let $(\Omega, \Im, P) = ([0,1], \mathcal{B}(\mathbb{R}) \cap [0,1], \mathcal{U}([0,1]))$. Let $\{X_n, n = 1, 2, \ldots\}$ be a sequence of random variables with $X_n \stackrel{d}{=} \mathcal{U}\left(\left[\frac{1}{2} - \frac{1}{n}, \frac{1}{2} + \frac{1}{n}\right]\right)$. Prove or disprove that $X_n \xrightarrow{d} X$ with $X = \frac{1}{2}$. (5 marks)