DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

MAJOR TEST 2015-2016 FIRST SEMESTER

MTL 107/MAL 230 (NUMERICAL METHODS AND COMPUTATION)

Time: 2 hours Max. Marks: 50

(**) This question paper has two parts: Part-A (for 40 Marks) and Part-B (for 10 Marks). Part-B is objective type. Attach Part-B to the main answer book. (**)

PART-A

**** Answer to each question should begin on a new page ****

1. Find the interval of unit length in which the smallest root of the equation $x^5 - x + 1 = 0$ lies. Taking midpoint of that interval as initial approximation perform one iteration of the second order Birge-Vieta method. (4)

2. Find leat square approximation of degree 2 for the function

x	0	1	2	3	4
f(x)	-4	-1	4	11	20

(4)

3. By use of repeated Richardson extrapolation , find $f'(1)$ from the following values:

x	0.6	0.8	0.9	1.0	1.1	1.2	1.4
f(x)	0.707178	0.859892	0.925863	0.984007	1.033743	1.074575	1.127986

Apply the approximate formula

$$f'(x_0) \simeq \frac{f(x_0+h) - f(x_0-h)}{2h}$$

with $h = 0.4, 0.2, 0.1$. (4)

4a. Derive two point Gauss-Chebyshev quadrature formula and hence evaluate

$$\int_{\frac{1}{2}}^{1} \frac{dx}{1+x}.$$

(5)

4b. Find the error in the three point Gauss-Legendre quadrature formula. $0, \pm\sqrt{\frac{3}{5}}$ (3)

5a. Using Crout's decomposition, solve the system of linear equations

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 5 \\ 3 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{matrix} 3 \\ -0.5 \\ 0.5 \end{matrix} \begin{bmatrix} 2 \\ 1 \\ 5.5 \end{bmatrix}.$$

1   -0.5 -0.5 $3 - 0.5 - 0.5 = 2$ $1 - 0.5$

(4)

5b. Let A and B be matrices of same order. Assume that A is non singular and suppose $\|A - B\| < \frac{1}{\|A^{-1}\|}$. Then find a bound on B^{-1} and hence Prove or disprove that

$$\|A^{-1} - B^{-1}\| \leq \frac{\|A^{-1}\|^2 \, \|A - B\|}{1 - \|A^{-1}\| \, \|A - B\|}.$$

(4)

6a. Assume that A is a strictly diagonally dominant matrix. Then prove or disprove that Gauss-Seidel iterations converge to the solution of $Ax = b$ for any initial(starting) vector $x^{(0)}$. (4)

6b. Find the optimal relaxation factor ω_{opt} if the following linear system is solved by Relaxation method.

$$\begin{array}{rcrcrcr} 4x & + & 0y & + & 2z & = & 4 \\ 0x & + & 5y & + & 2z & = & -3 \\ 5x & + & 4y & + & 10z & = & 2 \end{array}$$

(4)

7. Find the solution at the end of the first step of the Fourth order Runge-Kutta method in finding an approximation to the solution to the initial value problem

$$y' = 2x - y, \quad y(0) = -1$$

at $x = 1$ with $N = 10$. Here N denotes number of subintervals. (4)