# MTL107: NUMERICAL METHODS AND COMPUTATION
## MAJOR

Time: Two Hour

Total Marks: 50

1 **(5 Marks)** Assume that a smooth function $f$ has root $x^*$ which is $m$ times repeated. Show that, in general, Newton method will converge only linearly to $x^*$.

2. **(5 Marks)** Consider the following clamped cubic spline on $[0, 2]$

$$C(x) = \begin{cases} 1 + ax + 2x^2 - 2x^3, & \text{if } 0 \leq x \leq 1, \\ 1 + b(x-1) - 4(x-1)^2 + 7(x-1)^3 & \text{if } 1 \leq x \leq 2. \end{cases}$$

find $a, b, f'(0)$ and $f'(2)$.

3. **(5 Marks)** Perform two iterations of Steepest Decent method with initial guess $(0, 0, 0)^T$ for the following linear system of equations:

$$\begin{aligned} 4x_1 - x_2 + x_3 &= 2, \\ -x_1 + 6x_2 + x_3 &= 1, \\ x_1 + 2x_2 + 5x_3 &= 3. \end{aligned}$$

4. **(5 Marks)** Find the best fit quadratic polynomial to the following data:

| $x$ | 0 | 1 | 3 | 4 | 2 |
|---|---|---|---|---|---|
| $f$ | 3 | 1 | 4 | 3 | 0 |

5. **(5 Marks)** Consider the following inner product:

$$(f, g) = \int_0^\infty w(x) f(x) g(x) dx$$

with $w(x) = e^{-ax}$. Find first three orthogonal polynomials (zero, first and second degree) with respect to this inner product.

6. **(5 Marks)** State and prove Chebyshev Min-Max property for Chebyshev monic polynomials.

7. **(5 Marks)** Consider the following central difference formula approximating the derivative:

$$f'(x_0) \approx \frac{f(x_0 + h) - f(x_0 - h)}{2h}.$$

Assuming the roundoff error bound of $\epsilon$ find the optimal value of $h$ for minimum error.

8. **(5 Marks)** Find the order of following quadrature rule on interval $[-1,1]$:

$$\int_{-1}^{1} f(x)dx \approx \frac{1}{4}\left(f(-1)+3f(-1/3)+3f(1/3)+f(1)\right)$$

9. **(5 Marks)** Consider the following on step method for first order ODE:

$$y_{j+1} = y_j + h\phi(t_j, y_j, h),$$

where $\phi$ is Lipschitz continuous w.r.t. $y$ with Lipschitz contant $L$ and consistent. Assume that local truncation error is $\mathcal{O}(h^p)$, then show that:

$$||e_h(t_n)|| \leq \frac{Mh^p}{L}(e^{L(t_n-t_0)}-1).$$

Assume that initial error is 0.

10. **(5 Marks)** Consider the following initial value problem:

$$y' = -y^2 + t \qquad y(0) = 1,$$

Compute one iteration using Classical Runge-Kutta 4th order method with $h = 0.1$.