DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
MAJOR TEST 2022-2023 SECOND SEMESTER
MTL 107(NUMERICAL METHODS AND COMPUTATION)

Time: 2 hours

Max. Marks: 40

** **Answer to each question should begin on a new page.** **

****All notations are standard. Exhibit clearly all the steps to deserve full credit.****

"As a student of IIT Delhi, I will not give or receive aid in examinations. I will do my share and take an active part in seeing to it that others as well as myself uphold the sprit and letter of the Honour Code"

1a. If x, y, and z are machine numbers in a 32−bit word length computer, what upper bound can be given for the relative roundoff error in computing $z(x+y)$? (2)

1b. Consider Newton's method for finding the positive square root of $a > 0$. Then derive the following, assuming $x_0 > 0, x_0 \neq \sqrt{a}$.

$$x_{n+1}^2 - a = \left[\frac{x_n^2 - a}{2x_n}\right]^2,$$

$n \geq 0$, and thus $x_n > \sqrt{a}$ for all $n > 0$. Also prove that the iterates $\{x_n\}$ are a strictly decreasing sequence for $n \geq 1$. (4)

2a. A function $f(x)$ is defined on [0,1] and $| f^{(m)}(x) | \leq m!$ for $m = 1, 2, \ldots\ldots$ Let $P_n(x)$ be the interpolating polynomial of $f(x)$ at the points $1, q, q^2, \ldots\ldots, q^n$ where $0 < q < 1$. Then prove or disprove that $\lim_{n\to\infty} P_n(0) = f(0)$. (3)

2b. Determine a polynomial $p(x)$ of degree as low as possible such that

$$\max_{-1\leq x\leq 1} \left|\frac{1}{3+x} - p(x)\right| \leq 0.01$$

(Use Lanczos Economization). (4)

3. By use of repeated Richardson extrapolation, find $f'(1)$ from the following values:

x	0.6	0.8	0.9	1.0	1.1	1.2	1.4
f(x)	0.707178	0.859892	0.925863	0.984007	1.033743	1.074575	1.127986

Apply the approximate formula $f'(x_0) \simeq \frac{f(x_0+h)-f(x_0-h)}{2h}$ with $h = 0.4, 0.2, 0.1$. (4)

P.T.O.

4a. Derive two point Gauss-Chebyshev quadrature formula and hence evaluate

$$\int_{\frac{1}{2}}^{1} \frac{dx}{1+x}.$$

(5)

4b. Find the error in the three point Gauss-Legendre quadrature formula. (3)

5. Let A and B be matrices of same order. Assume that A is non singular and suppose $\|A-B\| < \frac{1}{\|A^{-1}\|}$. Then find a bound on B^{-1} and hence Prove or disprove that

$$\|A^{-1} - B^{-1}\| \leq \frac{\|A^{-1}\|^2 \, \|A-B\|}{1 - \|A^{-1}\| \, \|A-B\|}.$$

(4)

6a. Assume that A is a strictly diagonally dominant matrix. Then prove or disprove that Gauss-Seidel iterations converge to the solution of $Ax = b$ for any initial(starting) vector $x^{(0)}$. (4)

6b. Find the optimal relaxation factor ω_{opt} if the following linear system is solved by Relaxation method.

$$\begin{array}{rcrcrcr} 4x & + & 0y & + & 2z & = & 4 \\ 0x & + & 5y & + & 2z & = & -3 \\ 5x & + & 4y & + & 10z & = & 2 \end{array}$$

(4)

7. Find the solution at the end of the first step of the Fourth order Runge-Kutta method in finding an approximation to the solution to the initial value problem

$$y' = 2x - y, \quad y(0) = -1$$

at $x = 1$ with $N = 10$. Here N denotes number of subintervals. (3)