DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI
MINOR TEST I 2015-2016 FIRST SEMESTER
MTL 107/MAL 230 (NUMERICAL METHODS AND COMPUTATION)

Time: 1 hour — Max. Marks: 25

**** Answer to each question should begin on a new page ****

1a. Let $a = 1 \times 10^{308}$, $b = 1.01 \times 10^{308}$ and $c = -1.001 \times 10^{308}$ be three floating point numbers in F_D (IEEE double precision floating point system) expressed in their decimal form. Then find the values of $fl(a + fl(b + c))$ and $fl(fl(a + b) + c)$. (2)

1b. If x, y and z are numbers in F_s (IEEE single precision floating point system) expressed in their binary form, what upper bound can be given for the relative roundoff error in computing $fl(z \times fl(x + y))$, with rounding to the closest. (3)

2a. Determine the number of iterations required by **bisection method** to find the zero of $f(x) = x^3 - x^2 - 1$ on $[1, 2]$ with an absolute error of no more than 10^{-6}. (2)

2b. Consider a function f which satisfies the properties:
(i) There exists a unique root $\xi \in [0, 1]$;
(ii) For all real x we have $f'(x) \geq 2$ and $0 \leq f''(x) \leq 3$.
With initial approximation $x_0 = \frac{1}{2}$, how many iterations are required to get 10^{-6} accuracy by Newton-Raphson method? (4)

3. Using Sturm sequence, find the exact number of real roots of the equation

$$x^3 - 11x^2 + 32x - 22 = 0$$

lying in the interval $(3, 7)$. Perform one iteration of Newton-Raphson method to find the largest root of the above equation. (4)

4a. Assume $g(x)$ and $g'(x)$ are continuous for $a \leq x \leq b$, and assume g satisfies the property $a \leq x \leq b \Longrightarrow a \leq g(x) \leq b$. Further assume that $\lambda \equiv \text{Maximum}\,(| g'(x) |) < 1$ for all x in $[a, b]$. Then prove or disprove the following:
(i) There is a unique solution α of $x = g(x)$ in the interval $[a, b]$.
(ii) For any initial approximation x_0 in $[a, b]$, the iterates x_n generated by $x_{n+1} = g(x_n)$ satisfy

$$| \alpha - x_n | \leq \frac{\lambda^n}{1 - \lambda} | x_0 - x_1 | \ , \quad n \geq 0.$$

(4)

4b. For the following nonlinear system $4x_1^2 + 9x_2^2 - 36 = 0$, $16x_1^2 - 9x_2^2 - 36 = 0$ consider the fixed point method

$$x_1 = \phi_1(x) = \frac{1}{4}\sqrt{36 + 9x_2^2},$$

$$x_2 = \phi_2(x) = \frac{1}{3}\sqrt{36 - 4x_1^2}.$$

P.T.O.

Does it converge to the root $(1.8974, 1.5492)^T$ in some region around $(1.8974, 1.5492)$ with initial approximation $(1,1)^T$?. Justify your answer.

(3)

4c. Let $x_0, x_1, \ldots\ldots x_n$ be $n+1$ distinct points in $[a,b]$. Let $f_0, f_1 \ldots\ldots f_n$ be the values of $f(x)$ at these points. Then prove or disprove that for $k \geq 0$

$$f[x_n, \ldots\ldots, x_k] = \frac{1}{(n-k)!h^{n-k}} \nabla^{n-k} f_n.$$

(3)