DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI
MINOR TEST 2020-2021 FIRST SEMESTER
MTL107 (NUMERICAL METHODS AND COMPUTATION)

Time: 1 hour 10 minutes **Max. Marks: 30**

$**$ **Answer to each question should begin on a new page** $**$

1a. How many normalized numbers are there in the floating point system defined by

$$y = \pm\ .d_1 d_2 \ldots\ldots d_t\ \times\ \beta^e,$$

where each digit d_i satisfies $0 \leq d_i \leq \beta - 1$, $d_1 \neq 0$ and $e_{\min} \leq e \leq e_{\max}$? (2)

1b. If $\beta = 10, t = 4$ floating point arithmetic is used, what will be the unit roundoff for rounded arithmetic on machine. Also, find $fl(fl(1 + 0.0004))$. (2)

2a. If f is such that $\mid f''(x) \mid \leq 3$ for all x and $\mid f'(x) \mid \geq 1$ for all x, and if the initial error in the Newton-Raphson method is less than $\frac{1}{2}$, what is the upper bound on the error at each of the first three steps? (3)

2b. For the following nonlinear system $4x_1^2 + 9x_2^2 - 36 = 0, \quad 16x_1^2 - 9x_2^2 - 36 = 0$ consider the fixed point method

$$x_1 = \phi_1(x) = \frac{1}{4}\sqrt{36 + 9x_2^2},$$

$$x_2 = \phi_2(x) = \frac{1}{3}\sqrt{36 - 4x_1^2}.$$

Does it converge to the root $(1.8974, 1.5492)^T$ in some region around $(1.8974, 1.5492)$ with initial approximation $(1, 1)^T$?. Justify your answer. (3)

2c. The number of positive real roots of polynomial equation $P_n(x) = 0$ can not exceed the number of sign changes in $P_n(x)$ and the number of negative real roots of $P_n(x) = 0$ can not exceed the number of sign changes in $P_n(-x)$. Find the interval of unit length in which the smallest root of the equation $x^5 - x + 1 = 0$ lies. Taking midpoint of that interval as initial approximation perform one iteration of the second order Birge-Vieta method (Do calculations with four decimal places). (4)

3a. Consider the problem of finding a quadratic polynomial $p(x)$ for which $p(x_0) = y_0, \ \ p'(x_1) = y_1', \ \ p(x_2) = y_2$ with $x_0 \neq x_2$ and $\{y_0, y_1', y_2\}$ the given data. Assuming that the nodes x_0, x_1, x_2 are real, what conditions must be satisfied for such a $p(x)$ to exist and be unique?. (3)

3b. Let $x_0, x_1, \ldots\ldots x_n$ be $n + 1$ distinct points in $[a, b]$. Let $f_0, f_1 \ldots\ldots f_n$ be the values of $f(x)$ at these points. Then prove or disprove that for $k \geq 0$

$$f[x_n, \ldots\ldots, x_k] = \frac{1}{(n-k)!h^{n-k}} \nabla^{n-k} f_n.$$

(3)

3c. Let $f \in C^4[a, b]$. Let $x = a$ and $x = b$ be the nodes and $H(x)$ be Hermite interpolating polynomial of f . Then prove or disprove

$$\|f - H\|_\infty \leq \frac{(b-a)^4}{384}\|f^{(4)}\|_\infty.$$

(3)

4. Define

$$s(x) = \begin{cases} -\frac{11}{2}x^3 + 26x^2 - \frac{75}{2}x + 18, & 1 \leq \mathsf{x} \leq 2 \\ \\ \frac{11}{2}x^3 - 40x^2 + \frac{189}{2}x - 70, & 2 \leq \mathsf{x} \leq 3 \end{cases}$$

Examine whether $s(x)$ is a cubic spline function or not on [1,3]?. Is $s(x)$ a natural cubic spline function on [1,3]? Justify your answer. (3)

5a. Suppose $f^* = \sum_{j=0}^{j=n} c_j^* \phi_j$ be the least squares approximation to a given function f. Then TRUE or FALSE, justify the statement

$$\|f - f^*\|_2^2 = \|f\|_2^2 - \|f^*\|_2^2.$$

(2)

5b. Find the least squares straight line fit for $y = x\sqrt{1 - x^2}$ on $[-1, 1]$ using Chebyshev polynomials. (2)