MTL107: NUMERICAL METHODS AND COMPUTATION
MINOR I

Total Marks: 20 **Time: One Hour**

1. **(a) (3 Marks)** Find $PA = LU$ factorization of the following matrix using partial pivoting:

$$A = \begin{bmatrix} 2 & 1 & 5 \\ 4 & 4 & -4 \\ 1 & 3 & 1 \end{bmatrix}$$

where P is permutation matrix, L is lower triangular matrix with diagonal entries 1 and U is upper triangular matrix.

(b) (3 Marks) Prove that the following matrix is positive definite.

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Then determine its Cholesky Factorization.

2. **(4 Marks)** Let $f(x) = \sin^{(101)}(x)$. Consider the following formula for approximating the first derivative of f at $x = 1$:

$$f'(1) \approx \frac{f(1) - f(1-h)}{h}$$

If the maximum round-off error in approximating the values of f is ϵ, find the optimal value of h, for which the total error is minimum. Here, we assume that the total error is sum of round-off errors and truncation error (remainder term in Taylor's approximation Theorem.)

3. **(5 Marks)** Consider the linear system of equations,

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

Discuss the convergence of both Jacobi and Gauss Seidel methods. Use zero vector as initial guess and calculate two iterations of Jacobi and Gauss-Seidel Methods.

4. **(5 Marks)** Consider

$$\phi(x) = \frac{x^2 \cos(x)}{\sin(x) + x\cos(x)}.$$

The function ϕ has a fixed point at $x = \pi$. Prove or disprove the convergence of the fixed point iterations for this problem with suitable x_0. If the iterations converges, find the order of convergence.