# DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI
MINOR-I TEST 2022-2023 SECOND SEMESTER
MTL107 (NUMERICAL METHODS AND COMPUTATION)

**Time: 1 hour** **Max. Marks: 25**

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**** Answer to each question should begin on a new page ****
**** All notations are standard. Exhibit clearly all the steps to deserve full credit. ****

**1a.** How many normalized numbers are there in the floating point system defined by

$$y = \pm\ .d_1 d_2 \ldots\ldots d_t \ \times\ \beta^e,$$

where each digit $d_i$ satisfies $0 \le d_i \le \beta - 1$, $d_1 \neq 0$ and $e_{\min} \le e \le e_{\max}$? (3)

**1b.** If $\beta = 10, t = 4$ floating point arithmetic is used, what will be the unit roundoff for rounded arithmetic on machine. Also, find $fl(fl(1 + 0.0004))$. (2)

**2a. Write whether the following statements are TRUE or FALSE. No justification is required. Each question carries $\frac{1}{2}$ mark.**

In **IEEE** arithmetic, assume that $a$ and $b$ are normalized floating point numbers. Then
**(i)** $fl(a - b) = -fl(b - a)$
**(ii)** $fl(a + a) = fl(2 * a)$
**(iii)** $fl(0.5a) = fl(a/2)$
**(iv)** $fl((a + b) + c) = fl(a + (b + c))$ (2)

**2b.** If $x, y$ and $z$ are numbers in $F_s$ (IEEE single precision floating point system) expressed in their binary form, what upper bound can be given for the relative roundoff error in computing $fl(z \times fl(x + y))$, with rounding to the closest. (3)

**3a.** If $f$ is such that $| f''(x) | \le 3$ for all $x$ and $| f'(x) | \ge 1$ for all $x$, and if the initial error in the Newton-Raphson method is less than $\frac{1}{2}$, what is the upper bound on the error at each of the first three steps? (4)

**3b.** For the following nonlinear system $4x_1^2 + 9x_2^2 - 36 = 0, \quad 16x_1^2 - 9x_2^2 - 36 = 0$ consider the fixed point method

$$x_1 = \phi_1(x) = \frac{1}{4}\sqrt{36 + 9x_2^2},$$

$$x_2 = \phi_2(x) = \frac{1}{3}\sqrt{36 - 4x_1^2}.$$

Does it converge to the root $(1.8974, 1.5492)^T$ in some region around $(1.8974, 1.5492)$ with initial approximation $(1, 1)^T$?. Justify your answer. (3)

P.T.O.

**4.** Assume $g(x)$ and $g'(x)$ are continuous for $a \leq x \leq b$, and assume $g$ satisfies the property $a \leq x \leq b \Longrightarrow a \leq g(x) \leq b$. Further assume that $\lambda \equiv$ Maximum $(| g'(x) |) < 1$ for all $x$ in $[a, b]$. Then prove or disprove the following:

(i) There is a unique solution $\alpha$ of $x = g(x)$ in the interval $[a, b]$.

(ii) For any initial approximation $x_0$ in $[a, b]$, the iterates $x_n$ generated by $x_{n+1} = g(x_n)$ satisfy

$$| \alpha - x_n | \leq \frac{\lambda^n}{1-\lambda} | x_0 - x_1 | \ , \quad n \geq 0.$$

(4)

**5.** The number of positive real roots of polynomial equation $P_n(x) = 0$ can not exceed the number of sign changes in $P_n(x)$ and the number of negative real roots of $P_n(x) = 0$ can not exceed the number of sign changes in $P_n(-x)$. Find the interval of unit length in which the smallest root of the equation $x^5 - x + 1 = 0$ lies. Taking midpoint of that interval as initial approximation perform one iteration of the second order Birge-Vieta method (Do calculations with four decimal places).

(4)