Name: **Entry No:**

MTL107: NUMERICAL METHODS AND COMPUTATION
MINOR EXAMINATION

Total Marks: 40 **Time: 100 Minutes**

1. **(8 Marks)** Consider

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}.$$

Find the **LU** factorization of the matrix A. Write down all steps of factorization. Thereafter, use this factorization to solve the system $A\vec{x} = \vec{b}$ for $\vec{x} = [x_1\ x_2\ x_3]^T$.

2. **(8 Marks)** In which of the following intervals bisection method can be applied to find the zero of $f(x) = \sin(\pi x)$
(a) [1/4, 3/4] (b) [1/2, 5/4] (c) [-1/2, 2/3].
For each **acceptable** choice, estimate the number of iterations needed to ensure absolute error to be less than 10^{-3}. No marks will be awarded without proper justifications.

3. **(6 Marks)** Find the values of c for which the iteration $x_{n+1} = 2 - (1+c)x_n + cx_n^3$ converge to $x^* = 1$, provided x_0 is chosen sufficiently close to x^*. For what values of c will the convergence be quadratic?

4. **(6 Marks)** State the sufficient conditions for the convergence of Newton's root-finding method. Use these conditions to prove the convergence of the method.

5. **(6 Marks)** Prove that the strictly diagonally dominant matrices do not need pivoting for **LU** factorization.

6. **(6 Marks)** Find the **LDL**$^\top$ decomposition of the matrix:

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 3 \end{bmatrix}.$$

Use **LDL**$^\top$ decomposition to find the Cholesky's decomposition of A.

———————**Good Luck**———————

Question-1

Solution-1: $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & \cancel{l_{33}}\,1 \end{bmatrix}$ $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

$$LU = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+\cancel{l_{33}}u_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$$

$u_{11} = 1$ $\quad u_{12} = 2$ $\quad u_{13} = 4$

$l_{21}u_{11} = 3 \Rightarrow l_{21} = 3$ $\quad l_{21}u_{12} + u_{22} = 8 \Rightarrow u_{22} = 2$ $\quad l_{21}u_{13} + u_{23} = 14 \Rightarrow u_{23} = 2$

$l_{31}u_{11} = 2 \Rightarrow l_{31} = 2$ $\quad l_{31}u_{12} + l_{32}u_{22} = 6$, $4 + 2l_{32} = 6 \Rightarrow l_{32} = 1$ $\quad l_{31}u_{13} + l_{32}u_{23} + \cancel{l_{33}}u_{33} = 13$, $8 + 2 + u_{33} = 13$, $u_{33} = 3$

So, we get $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

for each entry $[9 \times 1/2 + 1/2]$ for clear solution — 5

* 5 marks if L,U are correct, partial marks if any entries are missing or any of the Matrix is wrong.

$Ax = b \Rightarrow LUx = b \rightarrow Ly = b$ where $Ux = y$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix} \Rightarrow \left.\begin{aligned} y_1 &= 3 \\ y_2 &= 4 \\ y_3 &= -6 \end{aligned}\right\}$$ By forward substitution $1\frac{1}{2}$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -6 \end{bmatrix} \Rightarrow \left.\begin{aligned} x_3 &= -2 \\ x_2 &= 4 \\ x_1 &= 3 \end{aligned}\right\}$$ By Backward substitution $1\frac{1}{2}$ — 3

* 3 marks if L & U are correctly used to find x. Partial marks if any of the entries are wrong.

Solution 2

$$\begin{bmatrix}1&2&4\\3&8&14\\2&6&13\end{bmatrix} \xrightarrow[R_3 = R_3 - 2R_1]{R_2 = R_2 - 3R_1} \begin{bmatrix}1&2&4\\0&2&2\\0&2&5\end{bmatrix} \xrightarrow{R_3 = R_3 - 1R_2} \begin{bmatrix}1&2&4\\0&2&2\\0&0&3\end{bmatrix}$$

l_{21}, l_{31}, l_{32}

$$L = \begin{bmatrix}1&0&0\\l_{21}&1&0\\l_{31}&l_{32}&1\end{bmatrix} = \begin{bmatrix}1&0&0\\3&1&0\\2&1&1\end{bmatrix} \text{ \& } U = \begin{bmatrix}1&2&4\\0&2&2\\0&0&3\end{bmatrix} \quad \text{— (5)}$$

5 marks if steps are mentioned & calculated correctly ~~and~~
Partial marks for wrong entries in L & U.

⇒ Next steps & marking is ~~same~~ same as solution 1.

Solution-3:

$$\begin{bmatrix}1&2&4\\3&8&14\\2&6&13\end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix}3&8&14\\1&2&4\\2&6&13\end{bmatrix} \qquad P_1 = \begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$$

$$L_1 = \begin{bmatrix}1&0&0\\1/3&1&0\\2/3&0&1\end{bmatrix} \text{ \& } L_1^{-1}P_1A = \begin{bmatrix}3&8&14\\0&-2/3&-2/3\\0&2/3&11/3\end{bmatrix}$$

$$P_2 = I, \quad L_2 = \begin{bmatrix}1&0&0\\0&1&0\\0&-1&1\end{bmatrix} \text{ \& } L_2^{-1}P_2L_1^{-1}P_1A = L_2^{-1}L_1^{-1}P_1A = \begin{bmatrix}3&8&14\\0&-2/3&-2/3\\0&0&3\end{bmatrix} = U \quad 3$$

$$L_2^{-1}L_1^{-1}P_1A = U \Rightarrow P_1A = \underbrace{L_1L_2}_{L}U \Rightarrow L = \begin{bmatrix}1&0&0\\1/3&1&0\\2/3&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&-1&1\end{bmatrix} = \underbrace{\begin{bmatrix}1&0&0\\1/3&1&0\\2/3&-1&1\end{bmatrix}}_{2}$$

[3+2] — (5)

$$Ax = b \Rightarrow P_1 A x = P_1 b \Rightarrow LUx = P_1 b = \begin{bmatrix}13\\3\\4\end{bmatrix}$$

$Ly = P_1 b$ where $Ux = y$

$$\begin{bmatrix}1 & 0 & 0\\ 1/3 & 1 & 0\\ 2/3 & -1 & 0\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix} = \begin{bmatrix}13\\3\\4\end{bmatrix} \Rightarrow \left.\begin{matrix}y_1 = 13\\ y_2 = -4/3\\ y_3 = -6\end{matrix}\right\} 1\tfrac{1}{2} \text{ forward substitution}$$

$$\begin{bmatrix}3 & 8 & 14\\ 0 & -2/3 & -2/3\\ 0 & 0 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}13\\ \cancel{3}-4/3\\ -6\end{bmatrix} \Rightarrow \left.\begin{matrix}x_3 = -2\\ x_2 = -4\\ x_1 = 3\end{matrix}\right\} 1\tfrac{1}{2} \text{ Backward substitution}$$

3

Solution 4:

$$\begin{bmatrix}1 & 2 & 4\\ 3 & 8 & 14\\ 2 & 6 & 13\end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix}3 & 8 & 14\\ 1 & 2 & 4\\ 2 & 6 & 13\end{bmatrix} \longrightarrow \begin{bmatrix}3 & 8 & 14\\ 2 & 6 & 13\\ 1 & 2 & 4\end{bmatrix}$$

$$P_1 = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}, \quad L_1 = \begin{bmatrix}1 & 0 & 0\\ 2/3 & 1 & 0\\ 1/3 & 0 & 1\end{bmatrix} \;\&\; L_1^{-1} P_1 A = \begin{bmatrix}3 & 8 & 14\\ 0 & 2/3 & 11/3\\ 0 & -2/3 & -2/3\end{bmatrix}$$

$$L_2 = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & -1 & 1\end{bmatrix} \;\&\; P_2 = I, \;\&\; L_2^{-1} L_1^{-1} P_1 A = \begin{bmatrix}3 & 8 & 14\\ 0 & 2/3 & 11/3\\ 0 & 0 & 3\end{bmatrix} = U \Big\} : 3$$

$$L_2^{-1} L_1^{-1} P_1 A = U \Rightarrow P_1 A = \underbrace{L_1 L_2}_{L} U \Rightarrow L = \begin{bmatrix}1 & 0 & 0\\ 2/3 & 1 & 0\\ 1/3 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & -1 & 1\end{bmatrix} = \underbrace{\begin{bmatrix}1 & 0 & 0\\ 2/3 & 1 & 0\\ 1/3 & -1 & 1\end{bmatrix}}_{2}$$

5

$$P_1 A x = P_1 b = \begin{bmatrix}13\\4\\3\end{bmatrix} \Rightarrow LUx = P_1 b \Rightarrow Ly = P_1 b$$

where $Ux = y$

$$\begin{bmatrix}1 & 0 & 0\\ 2/3 & 1 & 0\\ 1/3 & -1 & 1\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix} = \begin{bmatrix}13\\4\\3\end{bmatrix} \Rightarrow \left.\begin{matrix}y_1 = 13\\ y_2 = -14/3\\ y_3 = -6\end{matrix}\right\} 1\tfrac{1}{2} \text{ forward substitution}$$

$$\begin{bmatrix}3 & 8 & 4\\ 0 & 2/3 & 11/3\\ 0 & 0 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}13\\ -14/3\\ -6\end{bmatrix} \Rightarrow \left.\begin{matrix}x_3 = -2\\ x_2 = 4\\ x_1 = 3\end{matrix}\right\} 1\tfrac{1}{2} \text{ Backward substitution}$$

3

2. (8 Marks) In which of the following intervals bisection method can be applied to find the zero of $f(x) = \sin(\pi x)$

(a) $[1/4, 3/4]$ (b) $[1/2, 5/4]$ (c) $[-1/2, 2/3]$.

For each acceptable choice, estimate the number of iterations needed to ensure absolute error to be less than 10^{-3}. No marks will be awarded without proper justifications.

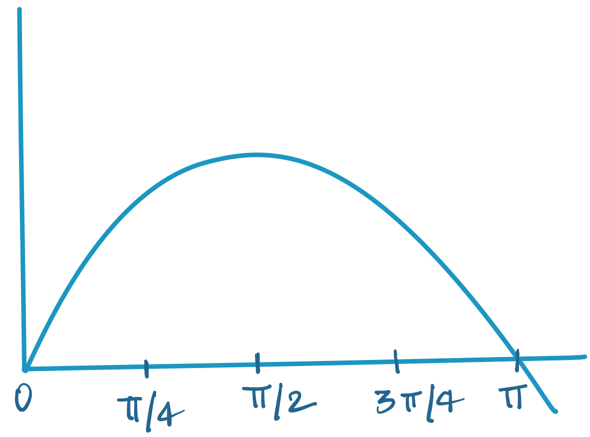
Bisection can be applied if : 1. $f(x)$ is Continous and defined on an interval $[a,b]$.

2. $f(a) \cdot f(b) < 0$ (1 Mark)

i.e $f(a)$ and $f(b)$ are of opposite Signs.

a). $f\left(\frac{1}{4}\right) = \sin\left(\frac{\pi}{4}\right) > 0$ $f\left(\frac{3}{4}\right) = \sin\left(\frac{3\pi}{4}\right) > 0$

(1 Mark)



or

$\sin(\pi x) > 0$ in $\left[\frac{1}{4}, \frac{3}{4}\right]$

or

No Roots lie in this Interval

So, Bisection cannot be applied.

b). $f\left(\frac{1}{2}\right) = \sin\left(\frac{\pi}{2}\right) > 0$ $f\left(\frac{5}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = \sin\left(\pi + \frac{\pi}{4}\right) < 0$

(1 Mark)

$f\left(\frac{1}{2}\right) \cdot f\left(\frac{5}{4}\right) < 0$

So, Bisection can be applied.

c). $f\left(-\frac{1}{2}\right) = \sin\left(-\frac{\pi}{2}\right) < 0$ $f\left(\frac{2}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right) > 0$

(1 Mark) $f\left(-\frac{1}{2}\right) \cdot f\left(\frac{2}{3}\right) < 0$

So, Bisection can be applied.

Now, Abs. Error $= \dfrac{b-a}{2^n}$

For Abs. Err $< 10^{-3}$ (2 Marks for correct Inequality)

# of Iterations : $\frac{b-a}{2^n} < 10^{-3}$ or $\frac{b-a}{2^{n+1}} < 10^{-3}$

i.e $n > \frac{\log\left(\frac{b-a}{\varepsilon}\right)}{\log 2}$

b). $n > \frac{\log\left(\left(\frac{5}{4} - \frac{1}{2}\right) \times 10^3\right)}{\log 2}$

(1 Mark)

$= \log\left(\frac{3000}{4}\right) / \log 2$

$= 9.55$

$=$ 10 Iterations [9 if $n+1$ is used].

c). $n > \log\left(\left(\frac{2}{3} + \frac{1}{2}\right) \times 10^3\right) / \log 2$

(1 Mark)

$= \log\left(\frac{7000}{6}\right) / \log 2$

$= 10.18$

$n = 11$ Iterations. [10 if $n+1$ is used].

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Q1 Question divided into 2 parts.

PART 1 Convergance.

~~$\|g'(x)\| < 1$~~

3 marks.
- Condition stated : $|g'(x)| < 1$: 1 mark
- g' correctly calculated : 1 mark
- Answer correctly given : 1 mark

PART 2: Quadratic convergance

Done in two ways.

I
- $g'(x^*) = 0$ & $g''(x^*) = 0$ 1 mark
- g' correctly computed : 1 mark
- Correct answer : 1 marks

II using definition

(i) $\|x_{n+1} - x^*\| < M \|x_n - x^*\|^2$ → 1 mark

(ii) Factorising $\|x_{n+1} - x^*\|$ after stating

2 marks given if this is done:

$$|x_{n+1} - x^*| = (x_n - 1)\,(\underbrace{c x_n^2 + c x_n - 1}_{x_n - 1 \text{ must be a factor}})$$

$\therefore c = 1/2$

4)

Statement : Let $f \in C^2[a,b]$. If $p \in (a,b)$ is s.t $f(p) = 0$ & $f'(p)$ then $\exists$ a $\delta > 0$. s.t Newtons method generates a seq. $\{p_n\}_{n=1}^{\infty}$ converging to p for an initial approximation $p_0 \in [p-\delta, p+\delta]$.

1

Proof. Suppose $p_n = g(p_{n-1})$ for $n \geq 1$ with

$$g(x) = x - \frac{f(x)}{f'(x)}$$

Let k be in $(0,1)$. We first find an interval $[p-\delta, p+\delta]$ that g maps into itself & for which

$|g'(x)| \leq k \quad \forall \; x \in (p-\delta, p+\delta)$.

Since f' is continuous & $f'(p) \neq 0 \Rightarrow$

$\exists \; \delta_1 > 0$ s.t $f'(x) \neq 0$ for $x \in [p-\delta_1, p+\delta_1] \subseteq [a,b]$.

Thus, g is defined & continuous on $[p-\delta_1, p+\delta_1]$.

Also,

$$g'(x) = 1 - \frac{f'(x)f'(x) - f(x)f''(x)}{[f'(x)]^2}$$

1

$$= \frac{f(x)f''(x)}{[f'(x)]^2}$$

for $x \in [p-\delta_1, p+\delta_1]$ and since, $f \in C^2[a,b]$, we have $g \in C^1[p-\delta_1, p+\delta_1]$.

By assumption $f(p) = 0$, so

$$g'(p) = \frac{f(p) f''(p)}{[f'(p)]^2} = 0 \quad (1)$$

since, g' is continuous & $0 < k < 1 \Rightarrow$ $\exists$ δ with $0 < \delta < \delta_1$ and

$$|g'(x)| \leq k \quad \forall \; x \in [p-\delta, p+\delta].$$

(1)

Now, to show that g maps $[p-\delta, p+\delta]$ into $[p-\delta, p+\delta]$. If $x \in [p-\delta, p+\delta]$ the Mean Value theorem implies that for some ξ b/w x and p

$$|g(x) - g(p)| = |g'(\xi)||x-p| \text{ so,}$$

$$\begin{aligned} |g(x) - p| = |g(x) - g(p)| &= |g'(\xi)| |x-p| \\ &\leq k|x-p| \\ &< |x-p| \end{aligned}$$

since $x \in [p-\delta, p+\delta]$, it follows that $|x-p| < \delta$ s.t $|g(x) - p| < \delta$.

(1)

Hence, g maps $[p-\delta, p+\delta]$ into $[p-\delta, p+\delta]$

All the hypotheses of fixed point theorem is satisfied.

So, the seq. $\{p_n\}_{n=1}^{\infty}$ defined by ①

$$p_n = g(p_{n-1}) = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})} \quad \text{for } n \geq 1$$

converges to p for any $p_0 \in [p-\delta, p+\delta]$.

When pivoting is not needed

Definition

A matrix $A \in \mathbb{R}^{n\times n}$ is diagonally dominant, if

$$|a_{ii}| \geq \sum_{k\neq i} |a_{ik}|, \quad i = 1, \ldots, n.$$

Theorem

If A is nonsingular and diagonally dominant then the LU factorization can be computed without pivoting.

Proof

We show that after reduction of the first row, the reduced system is again diagonally dominant. We have

$$a_{ik}^{(1)} = a_{ik} - \frac{a_{i1}a_{1k}}{a_{11}}, \quad i, k = 2, \ldots, n.$$

For the diagonal elements we get the estimate

$$\left|a_{ii}^{(1)}\right| = \left|a_{ii} - \frac{a_{i1}a_{1i}}{a_{11}}\right| \geq |a_{ii}| - \left|\frac{a_{i1}a_{1i}}{a_{11}}\right|, \quad i = 2, \ldots, n.$$

We now want to show that

$$\left|a_{ii}^{(1)}\right| \geq \sum_{k=2, k\neq i}^{n} \left|a_{ik}^{(1)}\right|, \quad i = 2, \ldots, n.$$

Proof (cont.)

Now for the sum of the moduli of the off-diagonal elements of row i (i = 2, . . . , n) we get

$$\begin{aligned}
\sum_{k=2,k\neq i}^{n} \left|a_{ik}^{(1)}\right| &= \sum_{k=2,k\neq i}^{n} \left|a_{ik} - \frac{a_{i1}a_{1k}}{a_{11}}\right| \\
&\leq \sum_{k=2,k\neq i}^{n} |a_{ik}| + \left|\frac{a_{i1}}{a_{11}}\right| \sum_{k=2,k\neq i}^{n} |a_{1k}| \\
&= \sum_{k=1,k\neq i}^{n} |a_{ik}| - |a_{i1}| + \left|\frac{a_{i1}}{a_{11}}\right| \left\{\sum_{k=2}^{n} |a_{1k}| - |a_{1i}|\right\} \\
&\leq |a_{ii}| - |a_{i1}| + \left|\frac{a_{i1}}{a_{11}}\right| \{|a_{11}| - |a_{1i}|\} \\
&= |a_{ii}| - \left|\frac{a_{i1}a_{1i}}{a_{11}}\right| \leq \left|a_{ii}^{(1)}\right|.
\end{aligned}$$

Question 6 minor

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$$\begin{bmatrix} 4 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$ 1st Part

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 2 \\ 0 & 5 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$ — 1 $R_2 = R_2 - \frac{1}{2}R_1$, $R_3 \to R_3 - \frac{1}{2}R_1$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ — 1

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/2 & 1/5 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9/5 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & 1 & 1/5 \\ 0 & 0 & 1 \end{bmatrix}}_{L^T}$$ $\left(R_3 \to R_3 - \frac{1}{5}R_2 \right)$

1 1

OR

Let $L = \begin{bmatrix} 1 & l_{12} & l_{13} \\ 0 & 1 & l_{23} \\ 0 & 0 & 1 \end{bmatrix}$ $d = \begin{bmatrix} d_1 & & \\ & d_2 & \\ & & d_3 \end{bmatrix}$

$$LDL^T = \begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix} \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \begin{bmatrix} 1 & l_1 & l_2 \\ 0 & 1 & l_3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} d_1 & 0 & 0 \\ l_1 d_1 & d_2 & 0 \\ l_2 d_1 & l_3 d_2 & d_3 \end{bmatrix} \begin{bmatrix} 1 & l_1 & l_2 \\ 0 & 1 & l_3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} d_1 & l_1 d_1 & l_2 d_1 \\ l_1 d_1 & l_1^2 d_1 + d_2 & l_1 l_2 d_1 + l_3 d_2 \\ l_2 d_1 & l_1 l_2 d_1 + l_3 d_2 & l_2^2 d_1 + l_3^2 d_2 + d_3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$ — 2

On comparison,

$d_1 = 4$

$l_1 d_1 = 2 \Rightarrow l_1 = 1/2$

$l_2 d_1 = 2 \Rightarrow l_2 = 1/2$

$l_1^2 d_1 + d_2 = 6 \Rightarrow \frac{1}{4} \times 4 + d_2 = 6$

$\Rightarrow d_2 = 5$ — 1 for atleast 4 correct evaluations.

$l_1 l_2 d_1 + l_3 d_2 = 2 \Rightarrow 1 + 5 l_3 = 2$

$\Rightarrow l_3 = \frac{1}{5}$

$l_2^2 d_1 + l_3^2 d_2 + d_3 = 3$

$\Rightarrow 1 + \frac{1}{5} + d_3 = 3 \Rightarrow d_3 = \frac{9}{5}$

$$\Rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/2 & 1/5 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9/5 \end{bmatrix}$$

1

2nd Part

If $A = L_1 L_1^T$ (Cholesky Decomposition)

Then $L_1 = L\sqrt{D}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/2 & 1/5 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & \sqrt{5} & 0 \\ 0 & 0 & 3/\sqrt{5} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 1 & \sqrt{5} & 0 \\ 1 & 1/\sqrt{5} & 3/\sqrt{5} \end{bmatrix}$$ — 1

OR

1 marks for finding correct cholesky decomposition with using LDL^T decomposition.