DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

MINOR TEST II 2015-2016 FIRST SEMESTER

MTL 107/MAL 230 (NUMERICAL METHODS AND COMPUTATION)

Time: 1 hour

Max. Marks: 25

** **Answer to each question should begin on a new page** **

1a. The interpolating polynomial for the function $f(x)$ on the set of distinct points $x_0, x_1, \ldots\ldots, x_n$ is given as $P_n(x) = \sum_{k=0}^{n} l_k(x) f(x_k)$. Find an explicit expression for $\sum_{k=0}^{n} l_k(0) x_k^{n+1}$. (2)

1b. A function $f(x)$ is defined on [0,1] and $\mid f^{(m)}(x) \mid \leq m!$ for $m = 1, 2, \ldots\ldots$ Let $P_n(x)$ be the interpolating polynomial of $f(x)$ at the points $1, q, q^2, \ldots\ldots, q^n$ where $0 < q < 1$. Then prove or disprove that $\lim_{n\to\infty} P_n(0) = f(0)$. (3)

2a. Suppose $f^* = \sum_{j=0}^{j=n} c_j^* \Phi_j$ be the least squares approximation to a given function f. Then prove or disprove $\|f - f^*\|_2^2 = \|f\|_2^2 - \|f^*\|_2^2$. (2)

2b. Approximate $f(x) = \sqrt[3]{x}$ by a straight line in the interval [0,1], in the least square sense with the weight function $w(x) = 1$. Also find the norm of the error function for the best approximation. (4)

2c. Find a polynomial of second degree which is the best approximation in maximum norm to $\sqrt{x}$ on the point set $\{0, \frac{1}{9}, \frac{4}{9}, 1\}$. (3)

3. For the method

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_1) - f(x_2)}{2h} + \frac{h^2}{3} f'''(\xi), \quad x_0 < \xi < x_2$$

determine the optimum value of h, using the criteria $\mid RE \mid = \mid TE \mid$. Using this method and the value of h obtained from the criteria $\mid RE \mid = \mid TE \mid$, determine an approximate value of $f'(2.0)$ from the following tabulated values of $f(x) = \log x$

x	2.0	2.01	2.02	2.06	2.12
f(x)	0.69315	0.69813	0.70310	0.72271	0.75142

given that the maximum roundoff error in function evaluation is 5×10^{-6}. (4)

4a. Find the number of subintervals n and the step size h so that the error for the composite Simpson's $\frac{1}{3}$ rd rule is less than 5×10^{-9} for the approximation $\int_2^7 \frac{dx}{x}$. (3)

4b. Compute

$$I = \int_0^1 e^{2x}\, dx$$

by Romberg integration method correct to three decimal places, using Trapezoidal rule. (4)