MTL107: NUMERICAL METHODS AND COMPUTATION

MINOR: 2

Total Marks: 20 **Time: One Hour**

1 (5 Marks) Find $PA = LU$ factorization of the following matrix using partial pivoting:

$$A = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 3 & -3 \\ 2 & 6 & 2 \end{bmatrix}$$

where P is permutation matrix, L is lower triangular matrix with diagonal entries 1 and U is upper triangular matrix.

2. (5 Marks) Prove that the following matrix is symmetric positive definite.

$$A = \begin{bmatrix} 9 & 6 & 3 \\ 6 & 8 & 6 \\ 3 & 6 & 14 \end{bmatrix}$$

Then determine its Cholesky Factorization.

3. (5 Marks) Consider the linear system of equations,

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

Use zero vector as initial guess and calculate two iterations of Jacobi and two iterations with Gauss-Seidel Method.

4. (5 Marks) Consider a function f witch take values, $(0, 3)$, $(1, -2)$, and $(2, 1)$. Use newton divided difference to find interpolating polynomial for the function. Furthermore, if f is smooth and $|f^{(3)}(x)| < K \ \forall x$, then estimate the maximum error.