$$| \alpha - x_n | \leq \frac{\lambda^n}{1-\lambda} | x_0 - x_1 | , \quad n \geq 0.$$

(4)

# DEPARTMENT OF MATHEMATICS

## INDIAN INSTITUTE OF TECHNOLOGY DELHI

## MINOR-II TEST 2022-2023 SECOND SEMESTER

## MTL107 (NUMERICAL METHODS AND COMPUTATION)

**Time: 1 hour** **Max. Marks: 25**

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**** Answer to each question should begin on a new page ****

**** All notations are standard. Exhibit clearly all the steps to deserve full credit. ****

**1a.** Consider the problem of finding a quadratic polynomial $p(x)$ for which $p(x_0) = y_0,\ p'(x_1) = y_1',\ p(x_2) = y_2$ with $x_0 \neq x_2$ and $\{y_0, y_1', y_2\}$ the given data. Assuming that the nodes $x_0, x_1, x_2$ are real, what conditions must be satisfied for such a $p(x)$ to exist and be unique?. (3)

**1b.** If the function $f$ is $k$ times continuously differentiable in the interval formed by the points $x_0, x_1 \ldots\ldots, x_k$, then prove or disprove that here is a point $\xi$ in this interval such that

$$f[x_0, x_1, \ldots\ldots, x_k] = \frac{f^{(k)}(\xi)}{k!}.$$

(3)

**2a.** Let $f \in C^4[a, b]$. Let $x = a$ and $x = b$ be the nodes and $H(x)$ be Hermite interpolating polynomial of $f$ . Then prove or disprove

$$\|f - H\|_\infty \leq \frac{(b-a)^4}{384} \|f^{(4)}\|_\infty.$$

(3)

**2b.** Define

$$s(x) = \begin{cases} -\frac{11}{2}x^3 + 26x^2 - \frac{75}{2}x + 18, & 1 \leq x \leq 2 \\ \frac{11}{2}x^3 - 40x^2 + \frac{189}{2}x - 70, & 2 \leq x \leq 3 \end{cases}$$

Examine whether $s(x)$ is a cubic spline function or not on [1,3]?. Is $s(x)$ a natural cubic spline function on [1,3]? Justify your answer. (3)

**3a.** Make a least squares fit for

| x | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| y(x) | 0.5 | 2 | 4.5 | 8 | 12.5 |

using the function $ax^b$ where $a$ and $b$ are constants. (4)

**3b.** Suppose $f^* = \sum_{j=0}^{j=n} c_j^* \phi_j$ be the least squares approximation to a given function $f$. Then TRUE or FALSE, justify the statement

$$\|f - f^*\|_2^2 = \|f\|_2^2 - \|f^*\|_2^2.$$

(2)

**3c.** Find the least squares straight line fit for $y = x\sqrt{1-x^2}$ on $[-1, 1]$ using Chebyshev polynomials. (2)

**P.T.O.**

4. Assume that f $\in C^5[a,b]$ and that $x-2h,\ x-h,\ x,\ x+h,\ x+2h \in [a,b]$. Further more if there exists a number $c = c(x) \in [a,b]$ so that

$$f'(x) = \alpha f(x+2h) + \beta f(x+h) + \gamma f(x-h) + \delta f(x-2h) + \text{Error},$$

(5)

where Error $=O(h^4)$. Find $\alpha, \beta, \gamma$ and $\delta$.